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The Method on Deducing Multiplicity Measurement Equations of Neutron/γ

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Abstract: Based on some important domestic and international references, the third γ multiplicity measurement equation is derived, but it is different from the results given in current researches. The neutron multiplicity equation is deduced in this paper, especially the fourth fast-neutron multiplicity equation based on the liquid scintillation detectors, which is more complex than the other multiplicity equations up to the fourth order. The equations given in this paper can be used to verify the validity and availability of principles for the multiplicity measurement up to the fourth order, and extend the application scopes of the neutron multiplicity measurement, such as correcting the additional measurement value to eliminate influences for dead times. It will be the foundation of nuclear researches, if the higher order multiplicity measurement is important for nuclear materials' control and accountability.

Keywords: neutron multiplicity, liquid scintillation detector, the multiplicity measurement, fast-neutron multiplicity.

1. Introduction

According to the point model assumption, the neutron multiplicity is related to the detection characteristics, detector parameters and basic nuclear constant. The neutron multiplicity measurement technique is one of the most advanced analytical techniques in NDA (nondestructive assay) technologies, which represents the latest development of NDA technology. NMC (Neutron Multiplicity Counter) technology is the inheritance and extension of the neutron coincidence measurement technique. For radioactive material with high density and non-uniformity distribution in airtight container, it can provide the quality attribute, oxidation degree etc. by the quantitative analysis of the neutron multiplicity distribution in nuclear sample fission.

The neutron multiplicity measurement is the key technique in the world, and some new methods [1-9] have been proposed for neutron multiplicity, which can improve the conditions of the traditional neutron multiplicity measurement, the measurement accuracy, the analysis ability of detecting the nuclear materials properties in airtight container as well as cut down the measurement time. Now researches of neutron multiplicity measurement are focused on principles, computer simulation and experimental system construction, so as to break through the bottleneck of technique development, and solve the authentication problem of nuclear components well. One of the key problems in the whole research process is the establishment of neutron multiplicity measurement equations [1-3], but in some domestic and international research reports and documents, the neutron multiplicity measurement equations up to the fourth order are directly given without detailed derivation. According to these methods, there are more parameters involved and they may result in mistakes easily.

Based on the principles proposed in literature [3], the third γ factorial moment is deduced in this paper, which is different from those in literature [1-3]. The reason is the later research cites the first one directly without deduction even if it is wrong. Moreover, the fourth fast-neutron multiplicity measurement equation [10-12] in different environment conditions are

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deduced in detail, which can not only verify the theory of the measurement equations up to the fourth order, but also improve the research abilities of neutron measurement, such as some parameters not requiring to be setted in advance, or additional measurement parameters being canceled.

2. The Third γ Factorial Moment

Based on the basic principles [3], including probability characteristic of spontaneous fission and induced fission, the probability function and multi-order derivative computation theory, the third γ factorial moment is deduced particularly, and one mistake is found in some important references, which is helpful to correct higher order factorial moments formulas.

In literature [1-3], the third γ factorial moment is

$$\langle\tilde{\mu}(\tilde{\mu}-1)(\tilde{\mu}-2)\rangle=\langle\mu_s(\mu_s-1)(\mu_s-2)\rangle$$
$$+3\langle\mu_s(\mu_s-1)\rangle\nu_s g_1+3\mu_s[\langle\nu_s(\nu_s-1)\rangle g_1^2+\nu_s g_2]$$
$$+\langle\nu_s(\nu_s-1)(\nu_s-2)\rangle g_1^3+3\langle\nu_s(\nu_s-1)\rangle g_2+\nu_s g_3$$

But the third γ factorial moment we derived is

$$\langle\tilde{\mu}(\tilde{\mu}-1)(\tilde{\mu}-2)\rangle=\langle\mu_s(\mu_s-1)(\mu_s-2)\rangle$$
$$+3\langle\mu_s(\mu_s-1)\rangle\nu_s g_1+3\mu_s[\langle\nu_s(\nu_s-1)\rangle g_1^2+\nu_s g_2]$$
$$+\langle\nu_s(\nu_s-1)(\nu_s-2)\rangle g_1^3+3\langle\nu_s(\nu_s-1)\rangle g_1 g_2+\nu_s g_3$$

Here, $3\langle\nu_s(\nu_s-1)\rangle g_2$ in references [1-3] is replaced with $3\langle\nu_s(\nu_s-1)\rangle g_1 g_2$. Why the mistake can not be found in recent years, it is believed that the factorial moment expression is too complex so that the result is cited directly, and the value of g_1 is so small that it is be ignored in experiments or simulations. But with the further improvement of measurement accuracy, it can bring some problems which are difficultly to understand.

3. The Fourth Neutron Multiplicity Measurement Equation

3.1 The Fourth Neutron (Plutonium) Multiplicity Measurement Equation

Based on six supposes and the neutron multiplicity measurement equations whose orders is up to 4, the fourth neutron (plutonium) multiplicity measurement equation is expressed as follows.

$$Q=\frac{F\varepsilon^4 f_q M^4}{24}\left\{\begin{matrix}\nu_{sf,4}+6\nu_{sf,3}\nu_{i2}\frac{M-1}{\nu_{i1}-1}+4\nu_{sf,2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}+15\nu_{sf,2}\nu^2{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2+\\ \nu_{sf,1}(1+\alpha)\frac{M-1}{\nu_{i1}-1}\left[\nu_{i4}+10\nu_{i2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}+15\nu^3{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2\right]\end{matrix}\right\}$$

Here, we let F is the sample spontaneous fission rate, F_α is (α, n) reaction rate in the sample, f_D and f_T represent the multiplicity distribution of the second and third factorial moment respectively, M is the material multiplication, α is the fraction of (α, n) reaction rate and spontaneous fission, ν_{sk} represents the k^{th} factorial moment of the multiplicity distribution of the spontaneous fission of neutron, ν_{sk} represents the k^{th} factorial moment of the multiplicity distribution of the induced fission of neutron, and k =1, 2, 3. S, D, T represent the three factorial moments of the multiplicity distribution of the leaking neutron respectively.

With the improvement of statistics and detector technique, the fourth neutron multiplicity measurement equation given above will be helpful to verify the results of the third order equation and simplify measurement process, such as reducing the set parameters number and correcting additional measurement value which is used to eliminate the affections of dead-time.

3.2 The Fourth Neutron (Uranium) Multiplicity Measurement Equation

Similar to the (plutonium) thermal neutron, the fourth neutron (uranium) multiplicity measurement equation is expressed as follows.

$$Q=\frac{\mathrm{F}\varepsilon^4 M^4 f_q\nu_{s4}}{24}\left\{\begin{matrix}1+6\nu_{s3}\nu_{i2}\frac{M-1}{\nu_{s4}(\nu_{i1}-1)}+4\nu_{s2}\nu_{i3}\frac{M-1}{\nu_{s4}(\nu_{i1}-1)}+15\frac{\nu_{s2}\nu^2{}_{i2}}{\nu_{s4}}(\frac{M-1}{\nu_{i1}-1})^2+\\ \nu_{s1}\frac{M-1}{\nu_{s4}(\nu_{i1}-1)}\left[\nu_{i4}+10\nu_{i2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}+15\nu^3{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2\right]\end{matrix}\right\}$$

4. The Fast Neutron Multiplicity Measurement Equation

4.1 The Fast Neutron (Plutonium) Multiplicity Measurement Equation

Different from the thermal neutron, the fast neutron measurement is not completely satisfied with the six assumptions of the traditional multiplicity measurement model, but always using the traditional equations in some references, these may lead to larger errors. To the fast-neutron multiplicity measurement, due to the presence of scattering crosstalk, the detector sometimes output two or even three neutron signals when a neutron coming. Therefore, we need to study the effect of the scattering crosstalk on the measurement data and put forward the appropriate hypothesis. Suppose that the situation can be ignored when a neutron makes more than two detectors response. And suppose that each neutron produces a signal in the first detector with the probability κ, and then produces a signal in the second detector, but does not make the third detector response. The probability generating function of producing the signal number based on a sample neutron emission should be

$$e(z)=(1-\varepsilon)+\varepsilon(1-\kappa)z+\varepsilon\kappa z^2$$

where ε represents the detection efficiency.

In the course of the derivation of the traditional neutron multiplicity equation, the distribution of the neutron multiplicity in the sample is not related to the detector. So the traditional methods can be used to deduce equations. Based on the probability generating function, the fast neutron (plutonium) multiplicity measurement equations based on liquid scintillation detectors are deduced as follows.

$$S=F\varepsilon(1+\kappa)\nu_{sf,1}(1+\alpha)M$$

$$D=\frac{F\varepsilon^2(1+\kappa)^2 f_d M^2}{2}\left[\nu_{sf,2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i2}\right]+F\varepsilon\kappa f_d\nu_{sf,1}(1+\alpha)M$$

$$T=\frac{F\varepsilon^3(1+\kappa)^3 f_t M^3}{6}\left[\nu_{sf,3}+\left(\frac{M-1}{\nu_{i1}-1}\right)3\nu_{sf,2}\nu_{i2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i3}+3\left(\frac{M-1}{\nu_{i1}-1}\right)^2\nu_{sf,1}(1+\alpha)\nu_{i2}^2\right]+F\varepsilon^2\kappa(1+\kappa)f_t M^2\left[\nu_{sf,2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i2}\right]$$

$$Quarters=\frac{F\varepsilon^4(1+\kappa)^4 f_q M^4}{24}\left\{\begin{array}{l}\nu_{sf,4}+6\nu_{sf,3}\nu_{i2}\frac{M-1}{\nu_{i1}-1}+4\nu_{sf,2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}+\\ 15\nu_{sf,2}\nu^2{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2+\\ \nu_{sf,1}(1+\alpha)\frac{M-1}{\nu_{i1}-1}\left[\begin{array}{l}\nu_{i4}+10\nu_{i2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}\\ +15\nu^3{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2\end{array}\right]\end{array}\right\}+\frac{F\varepsilon^3(1+\kappa)^2 kf_q M^3}{2}\left[\nu_{sf,3}+\left(\frac{M-1}{\nu_{i1}-1}\right)3\nu_{sf,2}\nu_{i2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i3}+3\left(\frac{M-1}{\nu_{i1}-1}\right)^2\nu_{sf,1}(1+\alpha)\nu_{i2}^2\right]+\frac{F\varepsilon^2k^2 f_q M^2}{2}\left[\nu_{sf,2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i2}\right]$$

4.2 The Fast Neutron (Uranium) Multiplicity Measurement Equation

The fast neutron (uranium) multiplicity measurement equation can be gained by analyzing the relationships between the neutron (plutonium) multiplicity measurement equations up to four and the neutron (uranium) multiplicity measurement equations up to four respectively. They are expressed as follows.

$$S=S_0+B+S_s+F\varepsilon(1+\kappa)\nu_{s1}M$$

$$D=\frac{F\varepsilon^2(1+\kappa)^2 f_d M^2}{2}\left[\nu_{s2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{s1}\nu_{i2}\right]+F\varepsilon\kappa f_d\nu_{s1}M$$

$$T=\frac{F\varepsilon^3(1+\kappa)^3 f_t M^3}{6}\left[\nu_{s3}+\left(\frac{M-1}{\nu_{i1}-1}\right)3\nu_{s2}\nu_{i2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{s1}\nu_{i3}+3\left(\frac{M-1}{\nu_{i1}-1}\right)^2\nu_{s1}\nu_{i2}^2\right]+F\varepsilon^2\kappa(1+\kappa)f_t M^2\left[\nu_{s2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{s1}\nu_{i2}\right]$$

$$Q=\frac{F\varepsilon^4(1+\kappa)^4 f_q M^4}{24}\left\{\begin{array}{l}\nu_{s4}+6\nu_{s3}\nu_{i2}\frac{M-1}{\nu_{i1}-1}+4\nu_{s2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}+\\ 15\nu_{s2}\nu^2{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2+\\ \nu_{s1}\frac{M-1}{\nu_{i1}-1}\left[\begin{array}{l}\nu_{i4}+10\nu_{i2}\nu_{i3}\frac{M-1}{\nu_{i1}-1}\\ +15\nu^3{}_{i2}(\frac{M-1}{\nu_{i1}-1})^2\end{array}\right]\end{array}\right\}+\frac{F\varepsilon^3(1+\kappa)^2 kf_q M^3}{2}\left[\nu_{s3}+\left(\frac{M-1}{\nu_{i1}-1}\right)3\nu_{s2}\nu_{i2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{s1}\nu_{i3}+3\left(\frac{M-1}{\nu_{i1}-1}\right)^2\nu_{s1}\nu_{i2}^2\right]+\frac{F\varepsilon^2k^2 f_q M^2}{2}\left[\nu_{s2}+\left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{s1}\nu_{i2}\right]$$

Here, all signs in the above formulas are the same with reference [3].

5. Conclusions

By researching the computation formula of multiplicity measurement in current literatures, the mistake of the third γ factorial moment is found, which is helpful to correct higher order moments. The fourth neutron measurement equation derived in this paper is different from literatures given by Los Alamos lab, but it is the same in essence. However, the fourth order measurement equation of fast neutron is derived firstly in this paper, which is important to improve and verify additional value in the front measurement computation. The equations given in this paper will be the foundation of nuclear material research, if the higher order multiplicity measurement is important for the measurement and accountability of nuclear material.

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