

# Formation Stiffness Measurement of a Train Track Interaction System

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**Abstract:** Railways are very important to our society due to their efficiency and reduced environmental effects. A system for the measurement of the condition of the formation on which a permanent way (pairs of rails laid on sleepers) is located is investigated in this work. This will allow effective asset management and reduce the costs of rail maintenance. Areas where the formation is either weak or changes rapidly present problems when maintaining a section of the track due to poor track geometry. Formation stiffness is a difficult parameter to measure and requires extensive research efforts. In this work a train-track interaction problem is investigated with a quarter train track model, which consists of a coach, bogie and wheel. The train-irregularity model is developed which computes the train response to irregularities, such as the deflection by stiffness changes. Using this train-irregularity model, the effects of train speed on the wheel/rail interaction force over the stiffness changes are studied and the track stiffness is also analysed, which will be used in future analysis to calculate the actual stiffness of the track when using laser measurement techniques.

**Key words:** Railway, train track, maintenance, stiffness.

## 1. Introduction

Balfour Beatty Rail and Heriot Watt University have established a development project supported by the Technology Strategy Board (TSB). The project is known as Formation Stiffness Measurement; formation stiffness variation has been identified as a possible underlying cause of poor and unstable track geometry. Balfour Beatty has developed a formation stiffness measurement concept specifically targeting the railway application, which involves a simple installation suitable for high speed measurement.

The formation stiffness measurement concept measures velocity by a laser Doppler vibrometer. Heriot Watt University will be in charge of modelling the behaviour of the system in the presence of cyclic, random and discrete track irregularities and non-linear stiffness, which will be addressed in this paper. Issues identified in the actual measurement analysis also include; track random irregularities, rail and wheel corrugation and discrete track defects. The prototype installation will be

on Balfour Beatty Rail's dynamic Track Stabiliser machine and testing on the Old Dalby test track will be used to evaluate performance.

Railway tracks near bridges or abutments, where railway track stiffness changes suddenly, often require additional maintenance in order to ensure good track geometry. The problems in the transition zone include ballast penetration into the subgrade, hanging sleepers, voiding, permanent rail deformation and cracking of concrete sleepers. Despite these problems, numerical studies of transition zones have been limited and further research is required to establish a theoretically consistent approach for designing the stiffness transition [1-4]. Transition zone stiffness will also be examined in this study.

A transition mechanism can be described as being at a point where the train wheels experience a rapid change in displacement due to a sudden change in the track stiffness. The change in displacement will excite the train components: bogie, coach and wheel, which will in turn leads to dynamically amplified vertical train

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track interaction forces [5, 6].

The formation stiffness measurement system to be researched provides a measurement of changes in the track modulus on which the permanent way is constructed. Track stiffness arises from different sources, such as the properties of the subsoil and ballast. The train-track interaction must be taken into account if a reasonable prediction of the track behaviour is to be made with varying stiffness of the track. The train track interaction system is modelled using a quarter train model and the analysis is solved using time domain solutions. This relatively simple approach is taken to allow the real time analysis from the laser vibrometer, i.e. to allow constant monitoring and analysis during actual track measurement.

The transition between different track stiffness, such as a track fault, is therefore being investigated in this work. A train-track irregularity model is developed which computes the train response to any given track fault. The effects of the transition type, such as a change between soft and stiff sides of a transition are examined. In addition, track stiffness changes due to sudden spikes (i.e. rail discontinuities) and continuous changes in track stiffness are also examined.

Railway track transition mechanics can be explained as follows: at transition zones the train wheels will

experience rapid changes in dynamic displacement (increased or decreased) caused by abrupt changes in track stiffness as shown in Figure 1. The level of dynamic excitation of the train and subsequent increased dynamic forces depend on the magnitude and frequency of the change, and the mass and suspension characteristics of the train. Even in cases of no significant dynamic load change in the transition zone, the ballasted track may eventually settle more than the track on the bridge abutment or tunnel base, creating dips in the track running surface and voided sleepers. The settlement in the vicinity of a transition zone can be significantly accelerated in the presence of soft subgrade soils and/or poor drainage. Because of the relationship between the dynamic train load and the track deflection, the railway track geometry in the transition zone can deteriorate at an accelerating rate through a self-perpetuating mechanism.

The analysis conducted in this work will be based on a track stiffness curve model which simulates the response of the train to a deflection change. This work is on-going and hence initial results are presented. The purpose of the developing model is to understand the effects of the train-track interaction on the new laser vibrometer system and hence how best to measure the track stiffness.

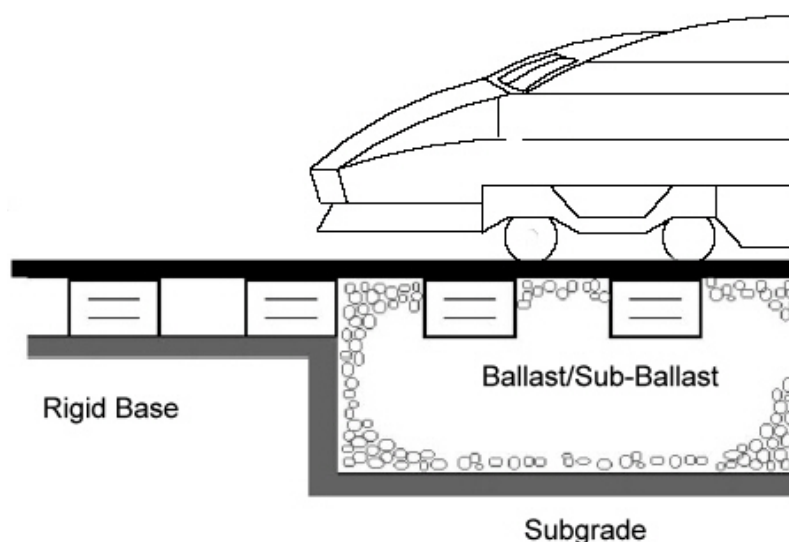


Fig. 1 A schematic layout of railway track stiffness transitions (Banimahd and Woodward, 2007).

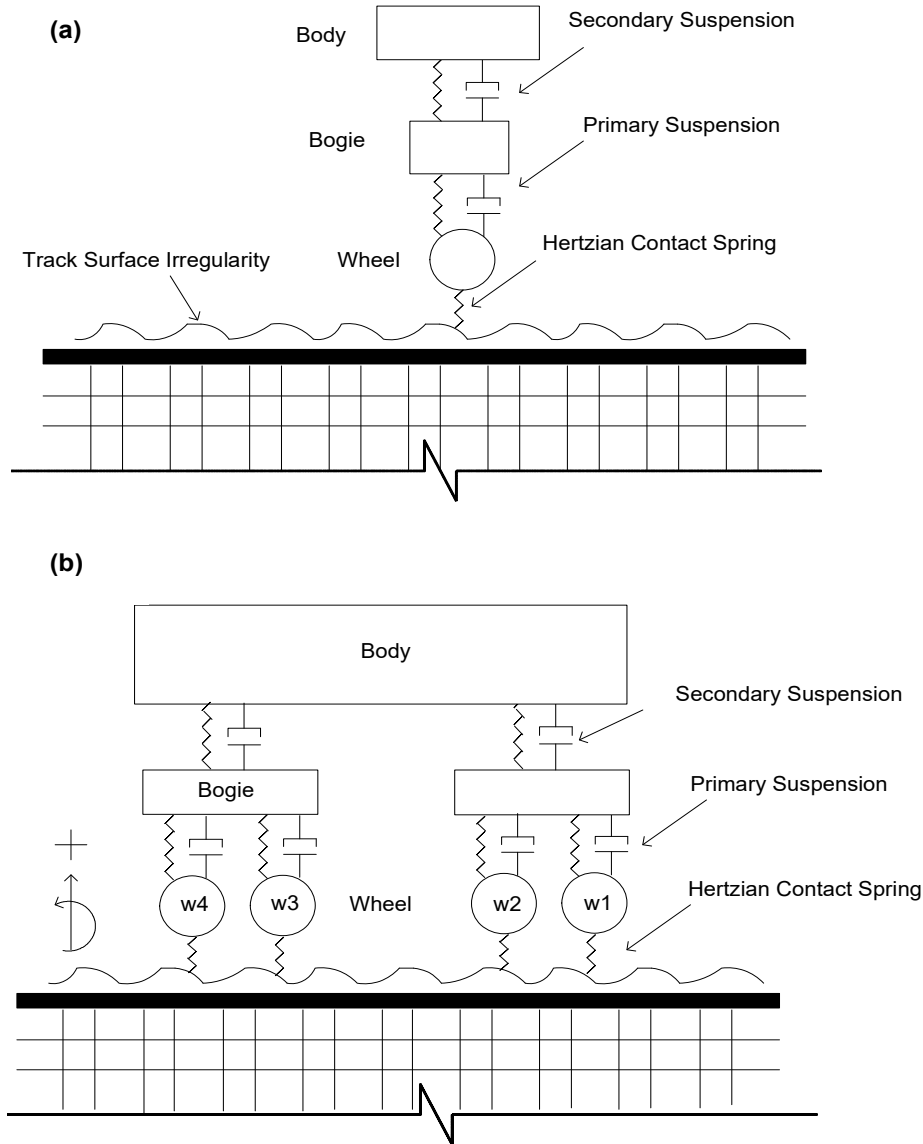


Fig. 2 Sketches of the (a) quarter carriage and (b) pseudo full carriage models.

## 2. Analysis

A sketch of the train interaction model is shown in Figure 2. The model comprises a quarter of a coach, half of a bogie and a wheel, which are interconnected through the suspension systems represented by dampers and springs. The symmetry of the carriage around the central track line is considered, and the model is formulated based on this. The system has three degrees of freedom, namely the wheel, the bogie and the coach vertical displacements. Considering the dynamic equilibrium, the equations of motion for the wheel, bogie and coach are formulated as follows.

Coach body bounce:

$$k_c(u_c - u_b) + c_c(\dot{u}_c - \dot{u}_b) + \frac{m_c}{8}g + \frac{m_c}{8}\ddot{u}_c = 0 \quad (1)$$

Bogie bounce:

$$k_b(u_b - u_w) + c_b(\dot{u}_b - \dot{u}_w) - k_c(u_c - u_b) - c_c(\dot{u}_c - \dot{u}_b) + \frac{m_b}{4}g + \frac{m_b}{4}\ddot{u}_b = 0 \quad (2)$$

Wheel bounce:

$$-F_{wr} - k_b(u_b - u_w) - c_b(\dot{u}_b - \dot{u}_w) + \frac{m_w}{4}g + m_w\ddot{u}_w = 0 \quad (3)$$

**Table 1** Specification of train used in this study.

	Mass(kg)	Stiffness (MN/m)	Damping (kN s/m)
Coach properties	9185	1.31	30
Bogie properties	815	3.28	90
Wheel properties	1000	93700	

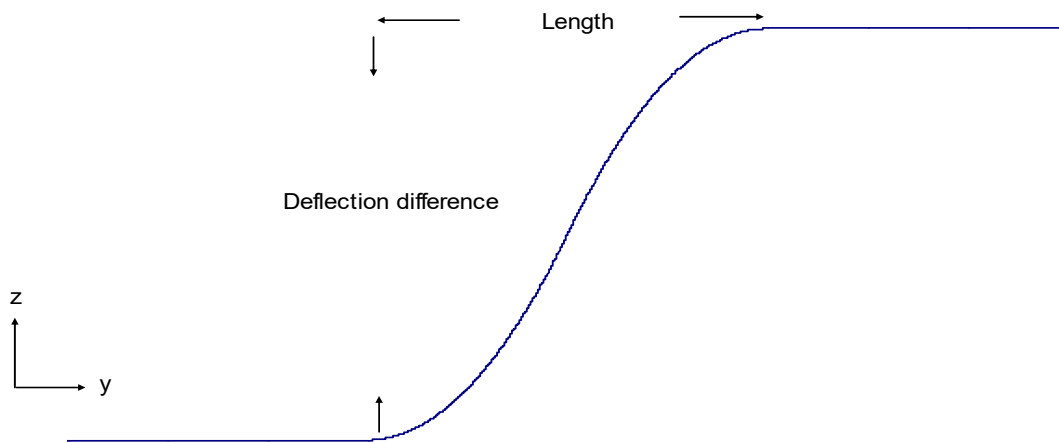
$u_w$ ,  $u_b$  and  $u_c$  are the vertical displacements of the wheel, bogie and coach,  $\dot{u}$  and  $\ddot{u}$  are the vertical velocity and acceleration of the train components respectively,  $m_w$ ,  $m_b$  and  $m_c$  are the wheel, bogie and coach masses respectively,  $k_b$ ,  $k_c$  and  $k_s$  are the stiffness coefficients respectively. Similarly  $c_b$ ,  $c_c$  and  $c_s$  are the damping coefficients respectively.  $F_{wr}$  is rail/wheel interaction force, which is modelled by the Hertz contact law:

These equations can be written in finite element format:

$$M\ddot{U} + C\dot{U} + KU = F_{ext} \quad (4)$$

$$\begin{aligned} U &= [u_w \quad u_b \quad u_c] \\ \dot{U} &= [\dot{u}_w \quad \dot{u}_b \quad \dot{u}_c] \\ \ddot{U} &= [\ddot{u}_w \quad \ddot{u}_b \quad \ddot{u}_c] \end{aligned} \quad (5)$$

$$\begin{cases} z(y) = R_{vert} \left\{ 1 - \sin\left(\arccos\left(\frac{y}{R_{vert}}\right)\right) \right\} & y \leq L/2 \\ z(y) = \frac{L^2}{4R_{vert}} - R_{vert} \left\{ 1 - \sin\left(\arccos\left(\frac{L-y}{R_{vert}}\right)\right) \right\} & y > L/2 \end{cases} \quad (10)$$

**Fig. 3** Stiffness transition representation as an irregularity.

$$M = \text{diag} \left[ m_w \quad \frac{m_b}{4} \quad \frac{m_c}{8} \right] \quad (6)$$

$$C = \begin{bmatrix} c_b & -c_b & 0 \\ \text{Sym.} & c_b + c_c & -c_c \\ & & c_c \end{bmatrix} \quad (7)$$

$$K = \begin{bmatrix} k_b & -k_b & 0 \\ \text{Sym.} & k_b + k_c & -k_c \\ & & k_c \end{bmatrix} \quad (8)$$

$$F_{ext} = \begin{bmatrix} -m_w g + F_{wr} \\ -\frac{m_b}{4} g \\ -\frac{m_c}{8} g \end{bmatrix} \quad (9)$$

### 3. Stiffness Transition

The deflection levels between the displaced/settled plain line track and the rigid base, the length in which this difference is traversed and the vehicle speed, are the factors which are studied here using a train-track irregularity model. The transition curve from the soft to the stiff base is produced using the following equations [7], as shown in Figure 3:

Where  $y$  is the distance along track in the transition zone,  $L$  is deflection spanning length and  $R_{vert}$  is vertical radius calculated from the following relationship:

$$R_{vert} = \frac{L^2}{4\Delta h} \quad (11)$$

where  $\Delta h$  is deflection difference between the track on the soft and stiff sides.

$$Stiffness(k) = \frac{force}{displacement} \quad (12)$$

There are three other different stiffness transition scenarios plotted below in Figures 4 and 5. In Figure 4 a continuous oscillating change in stiffness along the track is modelled, while Figure 5 a sudden spike, caused by a sudden change in track stiffness, is also modelled.

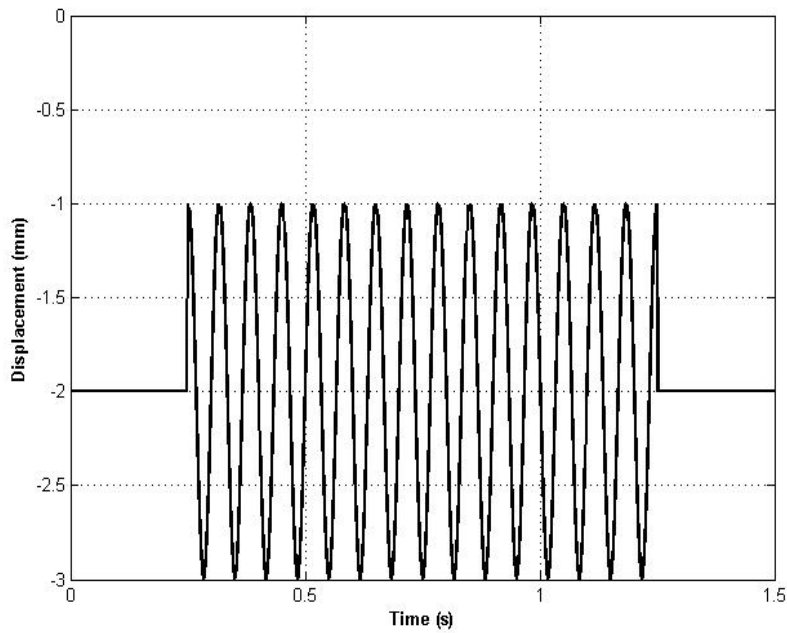


Fig. 4 Continuous cyclic oscillating stiffness along the track.

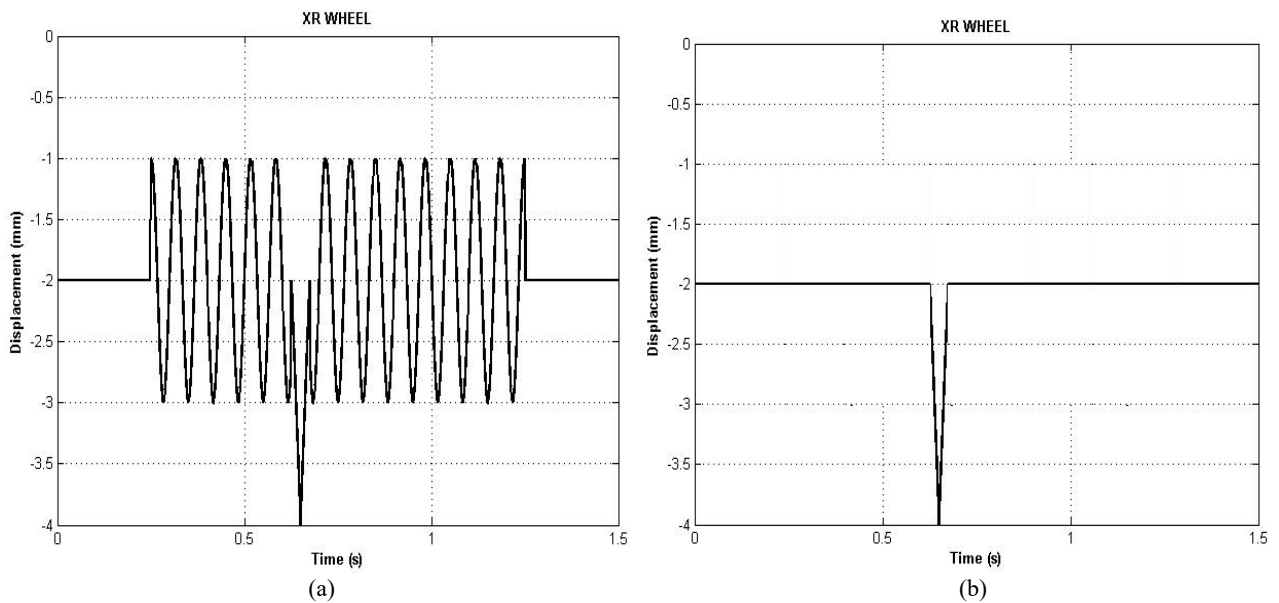


Fig. 5 a) Continuous cyclic stiffness change with a sudden spike, b) Sudden spike stiffness change.

### 4. Results

The train model is set up as follows; the train speed is set to 20 m/s and the affected length varies with different stiffness scenarios from 2 m for the spike to 20 m for the continuous cyclic test. The bogie part of the train is analysed and the results are shown in Figure 6 for various conditions. These results indicate the different behaviour of the train bogie when calculating the stiffness of the train track system. The effect of the track conditions and train damping is clearly observed.

The results shown in Figure 7 are plots of the bogie going over a transition zone with a continuously oscillating stiffness.

The results shown in Figure 8 demonstrate the train model going over a continuous transition change with a sudden spike. Figure 8a shows that there is a sudden velocity change when the train goes over the spike at 0.4s and also when it leaves the transition zone at 0.6s. Figure 8b shows a rapid acceleration over the spike and at the end of the transitions; these are the critical areas as there is a sudden change in stiffness.

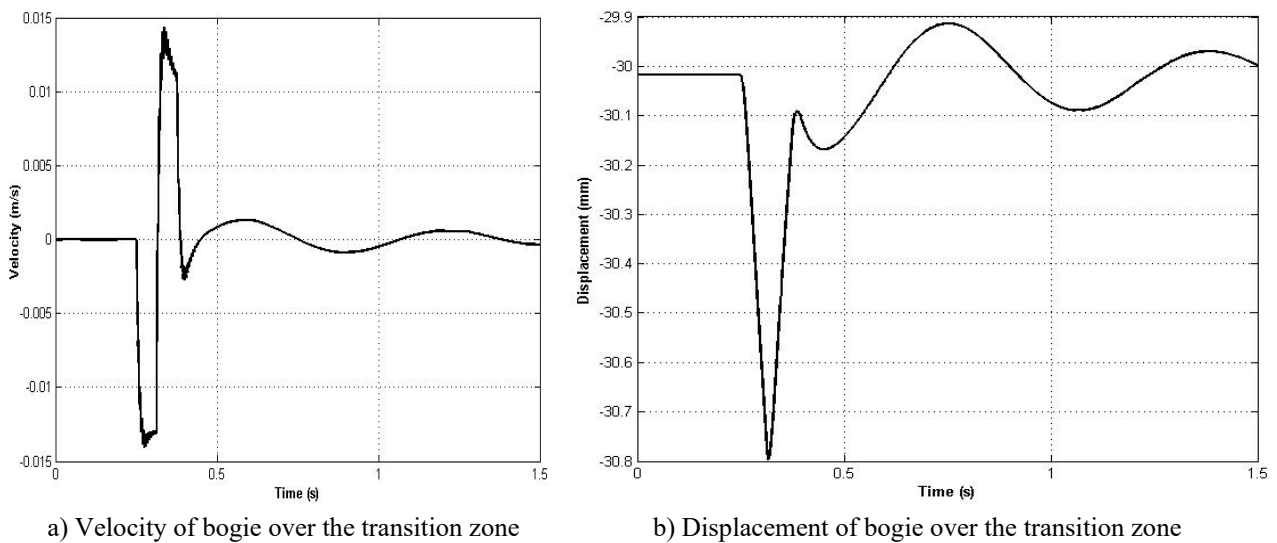
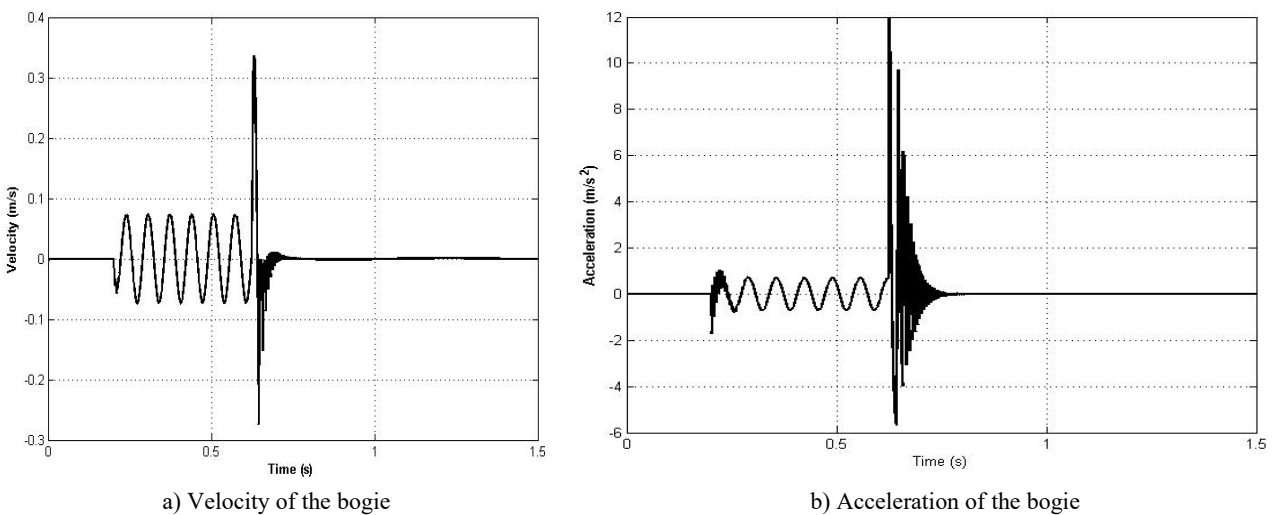
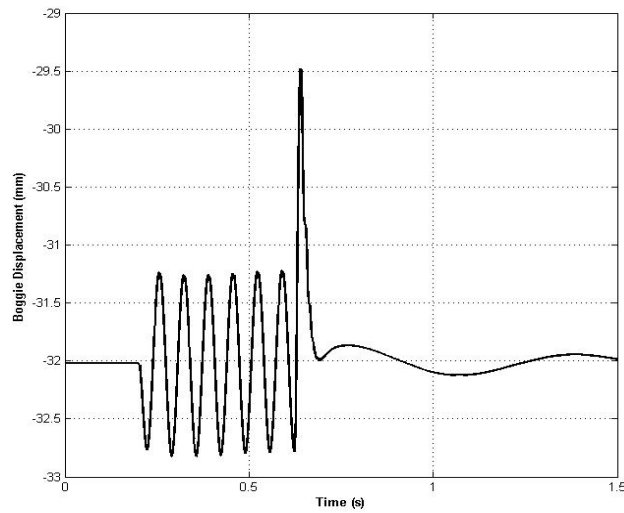


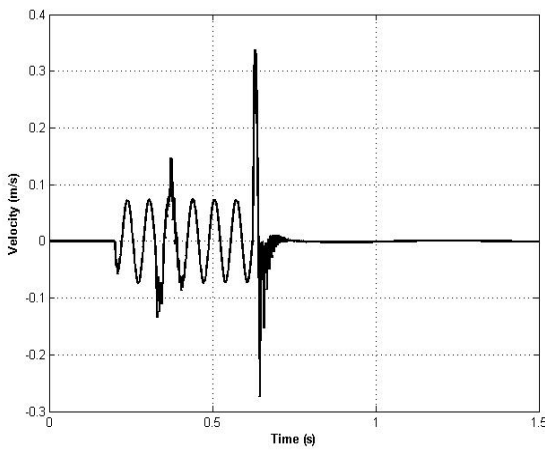
Fig. 6 Bogie response over a typical transition zone (velocity and displacement).



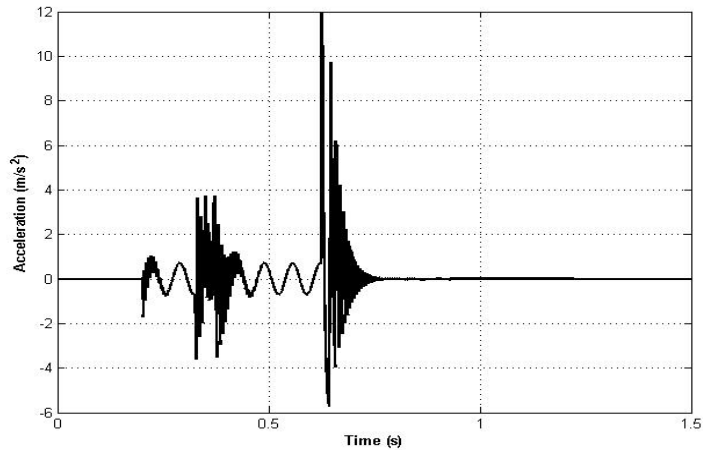


c) Displacement of the bogie

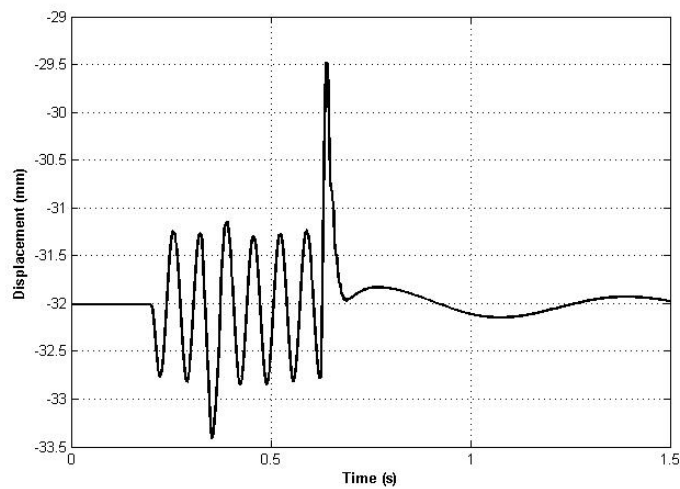
**Fig. 7** Response of a bogie going over a continuously oscillating stiffness change (velocity, acceleration and displacement).



a) Velocity of the bogie



b) Acceleration of the bogie



c) Displacement of the bogie

**Fig. 8** Response of a bogie going over a continuously oscillating stiffness change with a sudden spike (velocity, acceleration and displacement).

## 5. Conclusion

A train-track interaction problem was investigated in this preliminary work using a quarter train model which consisted of a bogie, coach and wheel. A simple train-track interaction model was developed which computes the train response to irregularities, such as the deflection due to stiffness changes. The model simulates the train excitation when passing over a transition curve representing a deflection change in the transition zone. Different transition scenarios of the train-track interaction system were investigated which included a sudden spike, a continuous stiffness change from soft to stiff soil and a continuous stiffness change. The results verify that the model is capable of detecting the bogie going over the various conditions. This analysis will be further developed to accurately calculate the stiffness of the train track system to allow real-time track stiffness determination from the laser vibrometer.

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