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Ninth-Grade Students' Difficulties in Solving Systems of Equations With Three Variables in UAE High Schools

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This study explores grade nine students' views about solving systems of equations with three variables in UAE high schools. To determine students' challenges and difficulties, the researcher conducted clinical interviews with five ninth-grade students with different performance levels (A, B, C, and D). The researcher interviewed each student individually and asked them about different difficulty levels, starting with easy questions and ending with difficult ones. After the interviews, the researcher presented the results using four themes: students' main struggles, students' emotions, students' choice of approach, and students' reflections. The researcher found that the students have difficulties solving linear equation systems with three variables related to basic arithmetic and algebraic skills and determining the best approach to solving the system of equations. Also, students felt anxious and unconfident when solving equations with three variables. The study results will help mathematics teachers develop instruction methods to support students' learning. The researcher provided some recommendations for mathematics teachers to improve their students' ability to solve systems of equations with three variables.

Keywords: ninth-grade, UAE, system of equations, linear equations, difficulties

Introduction

Mathematics is essential for counting, cooking, managing money, and building things. Beyond that, many career fields require a solid mathematical foundation, such as engineering, architecture, accounting, banking, business, medicine, ecology, and aerospace. Mullis and Martin (2016) reported that mathematics is vital to economics, finance, computing technology, and software development underlying our technologically advanced and information-based world. Mathematics enables learners to meet society's demands for an adequately qualified, flexible, or pliable workforce (Mushtaq, 2013).

Solving equations is essential since it helps students solve real-world problems algebraically. Students use symbols to represent the variables given in the problem; then, they write the equations to solve it (Chazan, Sela, & Herbst, 2012). Sometimes, solving linear equations is confusing for students. Students find three main difficulties when solving equations, as Magruder (2012) mentioned: first, symbolic understanding; second, the meaning of the equal sign; and third, reliance on procedural knowledge without conceptual understanding. Fatio, Fatimah, and Rosjanuardi (2019) reported that most students struggled to determine the solution of a system of linear equations with two variables algebraically, indicating that students could not select, use, and choose a specific procedure on this topic. In the presence of these difficulties, the role of mathematics teachers is to find

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methods to solve systems of equations. For example, Ives and Hoy (2003) used a graphic organizer to facilitate instructions in solving systems of equations with three variables.

There is evidence from national and international mathematics assessments that math difficulty, in general, and algebra, in particular, is a global issue. Wiberg (2019) mentioned that Swedish students' achievement in mathematics in Trends in Mathematics and Science Study (TIMSS) showed a decline in 2011. Comparing eighth-grade students' performance in TIMSS 2015 and TIMSS 2019 in the algebra domain it shows that some countries' averages declined. Those countries include Egypt, Finland, Hong Kong, Lebanon, Malaysia, Morocco, New Zealand, Singapore, the United States, and Abu Dhabi (United Arab Emirates) (Mullis, Martin, Foy, Kelly, & Fishbein, 2020). Also, many researchers, such as Jacobs and Spangenberg (2014), mentioned that South African students' performance in TIMSS was poor. In TIMSS 2011, South Africa's average in mathematics was the lowest compared with 21 medium-income countries. Several participating countries in PISA 2018 are concerned about this area since fewer students are pursuing future studies related to mathematics. At the same time, a growing demand exists for graduates in these fields (OECD, 2018).

Students need to understand mathematics lessons and achieve their learning outcomes. Also, students need to know their level in mathematics and measure their understanding and progress. According to the National Research Council (2001), mathematical proficiency has five strands: (1) conceptual understanding, which is the ability to comprehend mathematical operations, relations, and concepts; (2) procedural fluency, which means flexibility, accuracy, efficiency, and appropriateness in executing procedures; (3) strategic competence, which is the skill of formulating, representing, and solving mathematical problems; (4) adaptive reasoning, which is the ability to think logically, reflect, explain, and justify; (5) productive disposition, which is the habit of considering mathematics useful, helpful, and valuable, combined with a belief in one's ability and diligence.

In the UAE, the MOE schools use the Common Core State Standards; the standard that is related to solving systems of equations in three variables lesson is: "A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context" (www.corestandards.org). Figure 1 shows the lesson title and highland standards from the UAE's grade 9 elite scheme of work.

Week 5: Jan. 31 – Feb. 4, 2022		
Lessons	**Student Learning Outcomes**	**Common Core State Standards**
M9L7 – Optimization with Linear Programming	• Find maximum and minimum values of a functions over a region. • Solve real-world optimization problems by graphing systems of inequalities maximizing or minimizing constraints.	**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
M9L8 – Systems of Equations in Three Variables	• Solve systems of linear equations in three variables.	
M9L9 – Solving Absolute Value Equations and Inequalities by Graphing	• Solve absolute value equations. • Solve absolute value inequalities.	**A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Figure 1. The lesson title and standards (Source: Ministry of Education, UAE).

Solving systems of linear equations is considered one of the most challenging topics. Novianti and Prianta (2019) found that students experience difficulties when solving systems of linear equations in two variables due to not having mastered the material, having less understanding of linear equations in two variables, and having difficulties with arithmetic operations. Novianti and Prianta (2019) added that the lack of students' interest in

learning is another type of difficulty. If this is the case in solving systems of equations with two variables, solving systems of equations with three variables should be more complex and challenging for the students. Unfortunately, there is a literature gap on this topic. Still, from my experience as a mathematics teacher in high school, I found that students struggle to solve equations with three variables and make many mistakes.

Purpose of the Study and Research Question

The purpose of the study is to explore the difficulties in solving systems of equations in three variables with UAE ninth-grade students. The researcher investigated which methods the students prefer to use when they solve systems of equations and related problems. Moreover, the study will enrich the literature on this topic since no researchers have written about it in the context of the UAE. Also, the study aimed to provide some recommendations to help teachers improve their students' skills in solving systems of equations with three variables. This will help students overcome the difficulties and master learning goals related to the topic.

This Study has one research question: What difficulties do the UAE ninth-grade students experience when solving equations with three variables?

Significance of the Study

The importance of the study came from its role in revealing students' challenges in solving a system of equations with three variables. When students fail to solve problems related to the curriculum in the classroom, this affects their attitude toward the subject, which could affect students' performance and participation in mathematics activities. This study provides mathematics teachers with information about difficulties in solving systems of equations and suggests methods and resources for instructions. Also, the topic of the study is new, and there are no prior studies on this topic in the UAE.

Literature Review

Learning Difficulties in Mathematics

Unlike learning disabilities, students' learning difficulties are caused by outside factors or another student problem (Dumont, 1994). According to the DSF Literacy and Clinical Services in Australia, mathematical learning difficulties "dyscalculia" in secondary schools can be one of the following: (1) learning mathematical concepts outside primary number facts difficulties, (2) mental mathematics difficulties, difficulties in finding different ways in solving a mathematical problem, (3) delays in learning and recognizing mathematics vocabulary, (4) interpretation, and reading of maps, diagrams, and graphs difficulties, inability to perceive time as passing and difficulty adhering to schedules, a lack of budgeting skills, and spatial directions delays (DSF Literacy and Clinical Services, 2015). Also, many students make errors in mathematics because they lack the key vocabulary and are confused about symbols (Arnawa, Yerizon, & Nita, 2019).

When students lack adequate knowledge about a mathematical problem's features, they can attempt to make shallow surface strategies (Booth & Koedinger, 2008). Tambychik and Meerah (2010) stated that many factors might contribute to difficulties: lack of mastery of number facts, difficulty with computation, difficulty connecting conceptual aspects of math, inefficiencies in transmitting knowledge, difficulty making meaningful connections among information, inability to transform information mathematically, a lack of mastery of mathematical terms, a lack of understanding of mathematical language, and difficulty visualizing and comprehending mathematical concepts (Tambychik & Meerah, 2010). They added that it is likely that students'

inability to use cognitive abilities effectively to learn would affect their ability to acquire math skills. Steenbrugge, Valcke, and Desoete's (2010) study shows that mathematics education is a challenging subject during a pupil's entire primary school career. Furthermore, in primary school, some topics are considered more difficult than others, and some curriculum topics across all grades are described as challenging.

Learning Difficulties in Algebra

Algebra topics are essential in mathematics since success in algebra courses is considered a gateway to accessing advanced opportunities in mathematics and science (Booth, Lange, Koedinger, & Newton, 2013). Students entering Algebra 1 lack an accurate conceptual understanding of how equations work, negatively impacting their performance (Booth et al., 2013). Booth and Koedinger (2008) argued that students could not decide which strategy to apply when solving mathematical problems. As a result, they used the same strategy to solve different problems if it worked in some other problems. Also, Arcavi, Drijvers, and Stacey (2017) reported that many students make mistakes when they represent expressions algebraically; they use inappropriate signs. For example, when they translate "x more than eight," they write "8x" rather than "$x + 8$". They added that students struggle to make sense of algebraic problems (Arcavi et al., 2017).

Much research proved that most students in the early secondary grades (13 to 14 years old) often have difficulty with the algebra domain because of their previous poor experiences with arithmetic (Warren, Trigueros, and Ursini, 2016). Egodawatte (2009) found that about half of the students had "reversal error", which means students tried to translate the problem word by word or phrase by phrase to represent it algebraically. Egodawatte (2009) added that due to a poor understanding of the problem's arithmetic-algebraic connection, students could not understand the numerical relationship between the two quantities on an algebraic level. Jacobs and Durandt (2014) reported that most students have difficulty representing unknown values using variables. For example, they used two different variables to represent the same unknown value. According to Sulistiawati and Surgandini's (2019) findings, students enrolled in linear algebra courses have a weak level of self-confidence, which means that their confidence level is within normal limits. Yet, they tend to feel doubtful, worry about others' impressions, and think they are inferior.

Learning Difficulties in Linear Equations

Kieran (1981) and Baroody and Ginsburg (1983) stated that when solving equations, many students mistakenly believe that the equals sign means which side of the equation the answer belongs to, not an indicator of the balance between the two. Vlassis (2004) reported that students have difficulties with whole negative numbers. In his research in 2001, Vlallis found that the main challenges were reducing polynomials, simplifying two expressions on each side in the equation, or using the solution methods (Vlassis, 2001). Vlassis (2001) further added that understanding and using negative numbers depend heavily on the minus sign. Booth and Koedinger (2008) suggested that with misconceptions about how equals and negatives work, students do poorly on pretests and struggle to solve equations at the end of the lesson. Asquith, Stephens, Knuth, and Alibali (2007) reported that the National Assessment of Educational Progress results show that grade twelve students struggle to solve algebraic equations, even if they are simple and translated verbally to algebraic expressions. Also, their research findings show that many middle school students could not recognize equivalent equations before solving them. Students also struggle to determine the relationships between quantities and set-up equations (Asquith et al., 2007). Students may have conceptual or strategic difficulties in any of the following essential features in solving

problems algebraically: (1) dealing with and on known and unknown quantities, (2) proceeding solution alongside a group of logically related equations or inequalities, (3) unknowns identified and stable in the problem, (4) equation interpreted as formula, (5) narrative or an explanation of relationships between quantities, (6) intermediate quantities may not have a ready interpretation (Arcavi et al., 2017). Moreover, many students had difficulties in isolating variables when solving linear equations with one variable (Larino, 2018).

Ernawati and Muzaini (2020) asserted that when solving systems of equations with three variables, students made errors in reading, comprehension, transformation, process skill, encoding, and process errors. In the topic of solving a system of linear equations in two variables, Ernawati and Muzaini (2020) mentioned that the main difficulty in solving these types of equations is the concept difficulty, the problem of the students' incorrect reasoning about whether or not it is a system of linear equations in two variables, mistakes in writing the solution set and combining linear equations in two variables is not understood by students. Second, the principle is difficult, which arises from incorrect changes in data into standard form, incorrect changes in equations, and errors in completing the combined method using formulas in general. Third, there are difficulties with skills, such as miswriting the standard form, students' errors in writing questions when filling out answers, and operating numbers (Ernawati & Muzaini, 2020). Students made algebraic errors when solving linear equations because of mistakes in combining like terms, moving a variable to another side, and multiplying the equations by numbers (Issakova, 2006). Issakova (2006) added that students made mistakes in addition and subtraction when they solved linear equations.

Additionally, Luttenberger, Wimmer, and Paechter's (2018) study revealed that students with lower degrees of mathematics anxiety complete more linear equations correctly per minute and are more efficient than those with higher levels of mathematics anxiety. Birinci, Delice, and Aydın (2014) found that when students solve systems of linear equations, they prefer to use addition, subtraction, or multiplication operations rather than using new methods. Technology could support students' learning of algebra. For example, students can use calculators to check their answers after solving the equations (Hudson, Kadan, Lavin, & Vasquez, 2010). Also, calculators can be used to check calculations and graph functions in algebra (Ferrara, David, & Robutti, 2006). Furthermore, students' understanding of solving systems of linear equations could be improved by using real-life problems (Assadi & Hibi, 2022).

Theoretical Framework

Constructivism

Constructivism theory states that learners construct their knowledge by self-modifying their cognitive structure. Self-modification is an unconscious yet goal-directed process through which the individual changes how they think about a concept in reaction to a cognitive disturbance, relieving it (Faulkenberry & Faulkenberry, 2006). The student (ideally) thinks about a problem until it makes sense whenever they encounter a challenging problem (Faulkenberry & Faulkenberry, 2006). The constructivist theory is essential in learning mathematics since students must build their knowledge by actively involving them in learning. Students must understand what they already know and what they still need to remember for mathematics learning to be effective. Then, students need to be challenged and supported to learn more (Jazim, Anwar, & Rahmawati, 2017). Understanding algebra is the key to success in future mathematics courses, including geometry and calculus (Star et al., 2015).

Five E Model

Bybee (2009) mentioned that international mathematics education has extensively used the 5E model and tested students' gains in scientific process skills. In its history, the 5E model began with the question of "How People Learn", it has grown into an exemplary model for institutional pedagogy, especially in math and science education (Bybee, 2009). The model is called 5E because it consists of five stages, each starting with the letter E (Tezer & Cumhur, 2017). The following stages are included in the 5E learning cycle: *engaging* the students with a new concept is the teacher's first step in teaching the lesson, encouraging students to *explore* the idea or the skill, and giving an *explanation* of the results of the targeted concept, developing each idea or skill by *elaborating* it in additional practice, and *evaluating* their progress in a new setting as they proceed through the lesson (Turan & Matteson, 2021).

The researcher applies the 5E model by first engaging students in the problem and asking the students introductory questions such as: how many variables are there in the system? Then allow the students to explore how they solved the equations and which method they will use after comparing the different techniques. Then, the students will be asked to explain their ideas and the reasons behind using a particular technique. After that, students elaborate on some points or extended ideas. Finally, they evaluate their progress by looking back at their answers; in this step, students are asked to use an application or a scientific calculator to check their answers.

Methodology

This study applied a qualitative interpretive design. The interpretive approach allows the researchers to understand a variety of people's voices; it has a flexible nature and provides a profound description of participants' views, feelings, and experiences (Rahman, 2016). In this study, the researcher used clinical interviews with the students in the sample. Clinical interviews in mathematics assessment are essential since they are one-to-one (Lewis & Fisher, 2017). They allow the teacher to gather information about the students' knowledge and previous skills, which can help teachers support students' learning by improving instruction methods according to their needs (Heng & Sudarshan, 2013). The student can use any method to solve the assigned questions; they can write, draw, or use tables to represent their answers. The teacher observes the student and asks him/her probing questions depending on their answers (Lewis & Fisher, 2017).

The Study Participants

The sample is five ninth-grade elite-stream Emirati female students from a high school in Abu Dhabi. To represent grade 9 students (the population), the researcher selected participants from different academic mathematics levels and interests in mathematics learning.

The first two participants (student 1 and student 2) are 15 years old, and they are at level A (90-100) in mathematics and have a high interest in learning mathematics. They constantly engage in mathematics learning, participate in classroom activities and discussions, ask questions, and submit tasks on time. The third participant (student 3) is 15 years old, and she is of level B (80-89) in mathematics and has a moderate interest in mathematics learning. She engages in mathematics lessons, participates in activities, and asks questions whenever she is physically in the classroom. However, during the online lessons, her engagement is weak, her attendance is irregular, and sometimes, she delays the submission of tasks. The fourth student (student 4) is 14

years old and is at level C (70-80) in mathematics. She is interested in mathematics learning and engages in classroom activities while in the classroom, but in the online lessons, her engagement is weak. Also, sometimes, she could not work independently and asked for assistance from classmates. The fifth student (student 5) is 15 years old, and she is at level D (60-69) in mathematics. She attended all two-term lessons online, and her attendance was irregular. Also, her participation in mathematics lesson activities was weak, and she delayed the submission of tasks.

Clinical Interview Tools

The researcher used the question papers, a pen or pencil, and a calculator to conduct the clinical interviews. There were three lead questions in the question papers. All of them are from the grade nine mathematics textbook *Revel Math Integrated II* (McGraw Hill), and have different difficulty levels; then, there are a series of questions under each question. Clinical interviews are essential in numeracy projects because they enable teachers to understand children's thinking better as they work on a problem (Heirdsfield, 2002). In his study, Jenkins (2010) found that conducting clinical interviews helped prospective teachers find the right questions to ask to encourage children to clarify their thoughts. Moreover, interviews provide a unique opportunity to elicit the viewpoints of children who are usually silent in the classroom (McDonough, B. Clarke, & D. M. Clarke, 2002).

The first question's difficulty is low since the student can find the value of one variable (c) in one step using the first equation. Then, they will substitute the value of c in the second and third equations to turn them into a system of equations with two variables (see Figure 2).

1. System of equations with three variables.

$-2c = -6$

$2a + 3b - c = -2$

$a + 2b + 3c = 9$

a. How many variables are in the system of equations?

b. What methods can you use to solve the system of equations?

c. Which method do you prefer? Why?

d. Can you use the same method to solve any systems of equations? Why?

e. Solve the system of equations.

f. Why did you start with this step?

g. How can you check your answer?

Figure 2. Question 1, difficulty level: low.

The second question is more advanced than the first question. The first two equations have three variables, while the third has only two. The researcher added this question to see if the students would realize that they should solve the last equation for one variable first. Then, they will substitute it in the other equation or solve the system of equations by starting with the first equation (see Figure 3).

2. System of equations with three variables.

$$3x - 2y + 2z = -2$$

$$x + 6y - 2z = -2$$

$$x + 2y = 0$$

a. How many variables are in the system of equations?

b. What methods can you use to solve the system of equations?

c. Which method do you prefer? Why?

d. Can you use the same method to solve any systems of equations? Why?

e. Solve the system of equations.

f. Why did you start with this step?

g. How can you check your answer?

Figure 3. Question 2, difficulty level: medium.

Question 3 is more complex than the previous two questions since all three equations have three variables, and the student cannot solve one of them to substitute one variable in the other equations. To solve this system of equations, students must start with elimination, and this is the only system with no solution (see Figure 4).

3. System of equations with three variables.

$$f + 4g - h = 1$$

$$3f - g + 8h = 0$$

$$f + 4g - h = 10$$

a. How many variables are in the system of equations?

b. What methods can you use to solve the system of equations?

c. Which method do you prefer? Why?

d. Can you use the same method to solve any systems of equations? Why?

e. Solve the system of equations.

f. Why did you start with this step?

g. How can you check your answer?

h. What are the differences between questions 1, 2, and 3? Which one is easier and why?

i. Do you think solving equations with three variables is important? Why?

j. Can you find an example for using systems of equations in real life?

k. How can you improve yourself in solving systems of equations with three variables?

Figure 4. Question 3, difficulty level: high.

The Process

Since all participants were under 18, the researcher sent a letter to their parents to request their daughters' participation in the study. The researcher also informed the students and their parents that students' names would

not be mentioned in the study results. The students can withdraw from the study at any stage, and their participation is voluntary. After collecting permission from parents, the researcher invited the student to the library to conduct a 30-40-minute interview there since it is quiet and there are fewer distractions than in the classroom. Students can write their answers and discuss them with the teacher without interruption from others. The researcher observed the students while asking questions or answering them to gather data about students' feelings toward the questions. Also, the researcher saw which method the students used to solve the problems and then asked them to explain their solutions with follow-up questions.

Validity and Reliability

Researchers must be sensitive to validity and reliability issues, primarily when conducting qualitative studies (Brink, 1993). The research's validity relates to its findings' accuracy and truthfulness (LeCompte & Goetz, 1982). Generally, reliability refers to the consistency, stability, and repeatability of the informant's accounts and the investigator's ability to collect and record accurate information (Selltiz, Wrightsman, & Cook, 1976). The researcher used the following quality criteria to ensure the validity and reliability of the results: truthfulness, verisimilitude, authenticity, contextuality, and transferability. The researcher video-recorded each student interview using a smartphone camera to ensure truthfulness. The verisimilitude of the study is confirmed by reporting participants' emotions during the clinical interviews. The authenticity of the study came from its role in enhancing students' skills in solving systems of equations with three variables; that is, the study's results are helpful for mathematics teachers and curriculum and assessment departments in the UAE MOE. Also, there is a lack of research in this field; the researcher found some resources in algebra and systems of equations, but two variables and using different tools different from the clinical interviews used in this study. Contextuality of the study is guaranteed since all study parts are related to the same context (solving systems of equations with three variables) from the abstract to the conclusion. Also, the interview questions are about the same topic, and the participants have learned this topic in the same academic year. The transferability of the study results can be confirmed since the results can be generalized in the UAE and other countries with similar mathematics topics. Also, other researchers can use this study's results as a resource.

Data Analysis

The researcher adopted the thematic analysis to present the study results. In thematic analysis, "themes" are identified as patterns in the data that are important or interesting. These patterns can then be used to address the research or make a statement about an issue (Maguire & Delahunt, 2017). After completing the clinical interviews for all participants, the researcher classified them into four themes. The first theme is students' main struggles; it includes the difficulties they face when they solve the equations, which could be observed by the interviewer or reported by the students. The second theme is students' emotions, which presents students' feelings when they solve systems of questions. The third theme is the students' choice of approach; this theme displays the strategies that the students used in solving systems of equations with three variables. The fourth theme is students' reflections after the interview, such as using different methods, checking their answers, and improving themselves in solving systems of equations (see Figure 5).

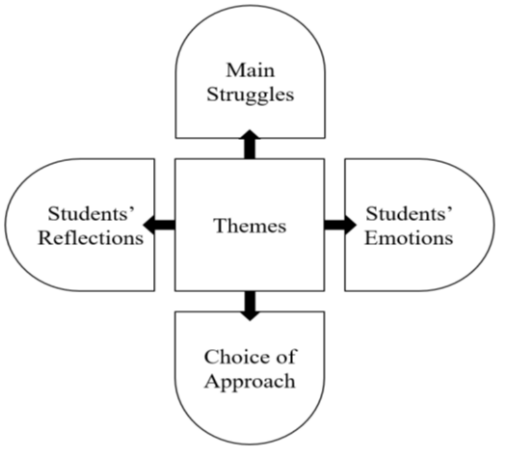


Figure 5. Themes of the findings.

Findings and Discussions

Students' Main Struggles

This theme focused on students' main errors when answering the interview questions, depending on the researcher's observations and students' answers.

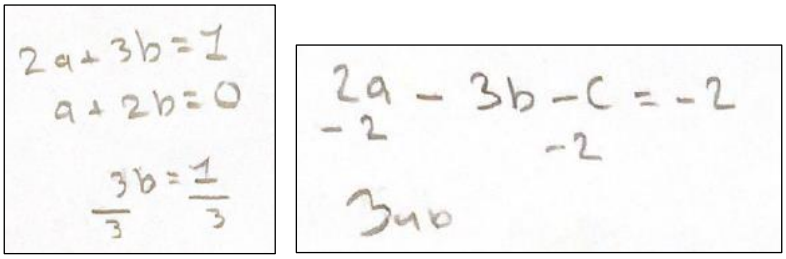
Student 1 answered questions 1 and 3 without any mistakes. However, in question 2, she mistakenly found the value of the first variable, "*y*". Then, when she substituted the value of "*y*" in the other equations to find the values of "*x*" and "*z*", she found the wrong values for them; that is, all of her answers were wrong.

Student 2 did not make mistakes in questions 1 and 3. In question 2, when she added the first two equations, she wrote the answer for -2 + -2 = -2, then the interviewer asked her to add them again, and she wrote the correct answer.

When student 3 answered question 1 first, she found the variable "*c*" value correctly. However, then she stopped and did not know what she should do. When the interviewer asked her to substitute the value of "*c*" in the other equations, she said that she did not know that she could replace it in the different equations, and she thought that the three equations related to each other. Also, she did not remember some essential vocabulary, such as substitution. When she wanted to say the elimination method, she said: "We put the two equations above each other." When she tried to eliminate a variable in two equations, she canceled one of the variables and wrote the other equal to the right side (see Figure 6).

Student 4 had difficulties in elimination; she did not know when to subtract the two equations directly and when to use multiplication and subtraction. Also, she had some errors in subtracting two equations; for example, when she subtracted "2*a* + 3*b* = 1" and "2*a* + 4*b* = 0", she wrote "*a* + *b* = 1" while the correct answer is "-*b* = 1", and when she wrote zero for the answer of "-3 – 3". When she started to solve the system of equations in the second question, she copied the first equation and then stopped. The interviewer asked the student: "What will you do?" she said: "I do not know." Then the interviewer gave her a hint by saying: "look at the second equation... What is the similarity between it and the second equation is? Will this help you solve the equation?" then, she realized that she should use elimination by addition to canceling the variable "*z*". When she solved the third system of equations she found that "0 = -9" which is the correct answer, but when the interviewer asked her what the meaning of her answer is, she said: "the equation is wrong," and she did not realize that there is no solution for the system.

Student 5 had difficulties in relating to basic skills such as solving equations of one step, understanding the variables, and how to add or subtract them. For example, in the first question, when she used the first equation, then she wanted to eliminate the variable "*a*" by subtracting "2" from the same side, and she wrote the answer "3*ab*" (see Figure 6b). In another step, when she substituted the value of "*c*" in the third equation by only writing it in its place, i.e., for "3*c*", she wrote "33" instead of multiplying "3" by "3". Also, when she multiplied to eliminate a specific variable, she multiplied the number by that variable without multiplying it by each term in the equation. Student 5's mistakes in questions 2 and 3 are similar to those in question 1. Moreover, the student needed assistance in each step and could not answer the questions independently without mistakes.



(a) (b)

Figure 6. (a) Student 3's mistake in the elimination; (b) Student 5's mistake.

From observing students' answers in solving the systems of equations, the researcher found that the student's difficulties in solving systems of equations with three variables came from errors and misconceptions in basic skills such as adding and subtracting integers from the observation. This confirmes Issakova's (2006) finding, who found that some students made mistakes in adding and subtracting numbers when solving linear equations.

Students have basic algebraic errors, such as solving one-step linear equations, combining like terms, and eliminating and subtracting variables. This finding is in line with Issakova's (2006) study which revealed that algebraic errors when solving linear equations could result from mistakes in combining like terms, moving a variable to another side, and multiplying the equation by a number. Larino (2018) found that students had errors in isolating variables when they solved linear equations with one variable.

Also, vocabulary errors are one source of difficulties that can hinder students from solving systems of equations. This finding confirms Arnawa et al.'s (2019) finding, which stated that mathematical errors are sometimes the result of a lack of technical vocabulary and confusion about the symbols. Similarly, it is in line with the DSF Literacy and Clinical Services (2015), which includes vocabulary as one of the difficulties in mathematics.

Students' Emotions

The interviewer asked the students about their feelings when they saw or answered a system of equations with three variables.

Student 1 said: "I feel stressed from the start until I solve it because any small mistake will affect my answer, which will be wrong. Also, when our teacher taught us this lesson, I told her that I did not want it to be included in the quiz."

When student 2 saw the questions before the interview started, she said: "I think it is difficult for me to answer them. I will go and review the lesson; then I will come tomorrow to answer them."

Student 3 said: “This is the most difficult lesson; I do not feel confident answering the questions. Also, sometimes I feel angry because I work to solve the system of equations for 15 minutes or more, then when I check my answer, I find it wrong, and I should repeat all steps to get the correct answer.”

Student 4 said: “When I see these types of questions, I feel sad because I know that my answer will be wrong or the time will not be enough to solve the system of equations. Also, while I am solving the system of equations with three variables, I feel confused, and I do not know from where I should start or which variable I will find first.”

Student 5 said: “When I see a system of equations with three variables, I get nervous. Actually, I do not know if these three equations are related to each other and what the relationship between the three variables is. I think I could not answer these questions without your support.”

From the students’ voices, the researcher concludes that the difficulties in solving equations with three variables made students feel anxious and unconfident in solving a system of linear equations with three variables. This finding is similar to Sulistiawati and Surgandini’s (2019) finding, which confirmed that in linear algebra courses, students have a low self-confidence level, which means that their confidence level is within normal limits, but they feel doubtful, worry about other people’s impressions, and think they are inferior to their classmates. Also, it is in line with the study of Luttenberger et al. (2018), which indicated that in contrast to those with higher levels of mathematics anxiety, students with lower levels of mathematics anxiety are more efficient at solving linear equations.

Choice of Approach

To know which approach the student uses to solve a system of equations with three variables, the interviewer asked them: “What method can you use to solve the system of equations with three variables? Which method do you prefer and why?”

Student 1 said: “I can use substitution, elimination, or graphing. However, I prefer to use substitution because I used to use it to solve systems of equations.”

Student 2 said: “I can solve a system of equations by elimination, substitution, or simplifying, but I prefer to use elimination because when I eliminate some variables, the system becomes easier to solve than before.”

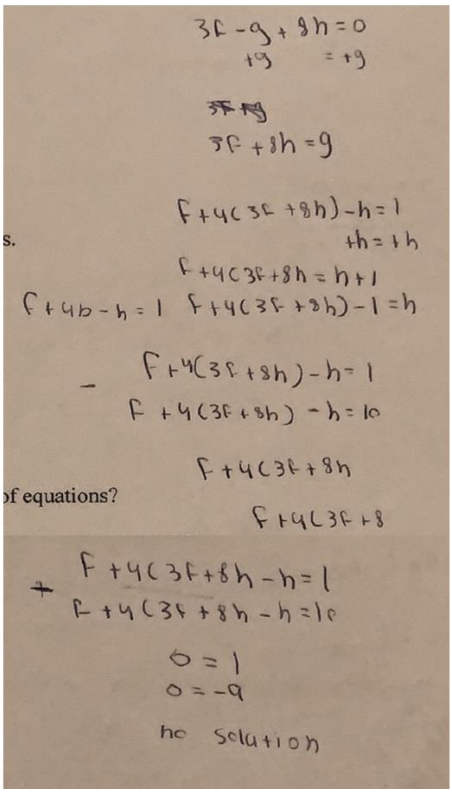
Student 3 said: “I do not remember the names of methods; I just start solving the equations in order, so I start with the first equation, then I put the equations above each other to subtract.”

Student 4 said: “I know the steps, but I do not remember the method’s name. When I solve a system of equations with three variables, I start with the easiest equation.”

Student 5 said: “I remember the graphing method; it is easy because when I graph the lines, I can see the solution in the graph, but for three variables, I cannot use the graphing method.”

From students’ voices, the researcher concludes that some students have misconceptions about solving systems of equations and difficulty in choosing the appropriate approach. For example, student 1 prefers to use substitution; however, elimination is easier for solving systems of equations with three variables. See Figure 6, which shows student 1’s answer and student 2’s answer. Student 1 took a long time and more steps than student 2, which means that by using the best approach with the appropriate equations, students can save time and effort in solving systems of equations. This finding is in line with Birinci et al. (2014), who stated that students prefer to use addition, subtraction, or multiplication operations rather than using new methods when solving systems of linear equations. Also, it confirms one of Vlassis’s (2001) findings, which showed that students had difficulty deciding which method to use to solve the equations.

Student 1 used substitution method



Student 2 used elimination method

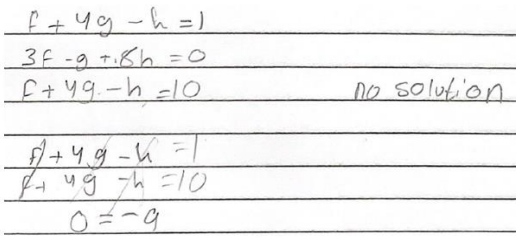


Figure 7. Students 1's and 2's solutions for question 3.

Students' Reflection

The interviewer asked the students: "What is the difference between question 1, question 2, and question 3?"

Student 1 said: "In question 1, I started the answer with elimination, but in questions 2 and 3, I started with substitution. Then I realized that it would be easier than substitution if I started question 3 with the elimination."

Student 2 said: "Question 1 was the easiest because I found '*c*' directly from the first step. In question 2, I used elimination. In question 3, there is no solution; also, I realized that if I multiplied the second equation by 2, it would take a longer time, but I will find the same solution."

Students 3 and 4 said: "In question 1, one of the equations was with one variable and the other two equations had three variables. One equation had two variables in the second question, while the other two had three. In the third question, all three equations had three variables."

Student 5 said: "I think question 2 is more straightforward than question 1 because when I answered question 1, I practiced solving systems of equations, then it became easier for me to solve the second question. Question 3 is different from them because it has no solution."

From the previous dialogs, the researcher found that some students did not understand the difference between the three questions and had a different understanding of them. Students 3 and 4 described the difference between the three questions better than the other three students. Mevarech and Kramarski (2003) found that asking students reflexive questions about the similarities and differences between questions they solved enhances their conceptual knowledge and procedural knowledge.

Then the interviewer asked the students: How can you check your solution?

Students 1 and 4 said: "I used substitution."

Student 2 said: "By substitution, but if I want to save time, I use online applications such as Mathway and cymath."

Student 3 said: "I substitute my solutions in the equations, and sometimes I use a calculator."

Student 5 said: "I use substitution, but also I can use the calculator by using the mode button, and sometimes I use computer applications such as Mathway."

From students' answers, the researcher found that all students checked their answers by substituting the value of each variable in each equation in the system, which is the traditional method of checking solutions. Checking answers by substituting takes time, and students could make some mistakes while checking. Some students use technology to check their answers, such as calculators or online computer applications, saving time and giving accurate answers. In their research, Hudson et al. (2010) mentioned that students checked their answers using a calculator. Similarly, Ferrara et al. (2006) stated that calculations and graphs of functions could be checked using technology.

The interviewer asked: "Do you think solving equations with three variables is important and why?"

Student 1 said: "I think it is important because I remembered that we had some real-life examples when we learned this lesson."

Student 2 said: "It is important because we can use it. For example, if someone buys three items with different prices, we can apply a system of equations with three variables."

Student 3 said: "I think it is important because I can use it. For example, if I were an engineer, it would be useful if I knew how to solve systems of equations."

Students 4 and 5 said: "I think it is important, but I do not know why."

From the previous conversations, the researcher found that some students understand the importance of solving systems of linear equations with three variables in their lives, while others do not. Assadi and Hibi (2022) mentioned that solving linear equations using real-life situations has a notable advantage. Integrating real-life experiences into a school subject lends credibility and malleability to the subject, which benefits students and educators alike.

The interviewer asked: "How can you improve yourself in solving systems of equations with three variables?

Student 1 said: "I keep practicing, solving equations, and checking my solutions."

Student 2 said: "I practice more in solving systems of equations with three variables. Also, sometimes I use applications that can show me the solution step by step."

Student 3 said: "I practice by answering extra questions from the student book, and I watch videos to remember the steps."

Students 4 and 5 said: "I practice using examples that my teacher provides."

All students reported that they practice solving more systems of equations with three variables to improve their skills and ability to these systems of equations. Also, some students mentioned that they used online mathematics applications or videos to improve themselves in solving systems of equations. This means students need training in solving systems of linear equations to develop their skills to master this topic, and this training could be done through written examples or videos and mathematics applications. Hsiao, Lin, Chen, and Peng (2017) mentioned that using training systems with students positively impacts their performance.

To sum up, the results of the clinical interview showed that students have difficulties in solving systems of equations with three variables related to their experience in mathematics, basic knowledge, and skills in numbers and algebra. These difficulties made the students feel anxious and unconfident when they solved these types of equations. Also, unknowing the best approach to solve the system of equations or from where they start is another difficulty.

Most of the participants' mistakes in solving systems of equations with three variables are the results of weaknesses in basic mathematics skills learned in previous years. These results confirm that the constructivism theory is essential in mathematics learning, as mentioned in the theoretical framework. Also, the results of clinical interviews emphasize the role of the 5-E model in learning mathematics. The interviewer started the interview with general questions about the title and the equations to *engage* the students in the questions. The students started to *explore* the system of equations and think about how they could solve it. Then, the interviewer asked the students to *explain* how they solved it. They *elaborate* to solve the second and third systems. After they answered all questions, the researcher asked them to *evaluate* themselves and reflect on their answers using a series of questions.

Implications and Limitations

From the previous findings, the researcher suggests some recommendations for mathematics teachers to help their students overcome difficulties and improve their performance in solving systems of equations with three variables. Firstly, teachers are encouraged to conduct a diagnostic test on the prior experience of their students at the beginning of the year to find out the weaknesses or misconceptions related to their basic skills in numbers or algebra areas, then guide their students to practice in these areas. Secondly, teachers should ensure that their students know the critical vocabulary needed in their lessons. Teachers can improve their students' mathematics vocabulary by using games or flashcards at the start. Thirdly, teachers can improve their students' confidence in solving systems of equations with three variables by implementing activities such as group work and nominating them to explain their answers to their classmates. Fourthly, when teachers introduce the lesson, it is essential to ask the students to compare the systems and the equations in each system and find the most appropriate strategy to solve each system. Fifthly, teachers must teach their students to check their solutions by different methods such as substitution if they are not allowed to use calculators. Also, students need to know the importance of checking their solutions after solving them. Sixthly, teachers are advised to prepare resources such as extra practice worksheets, videos, or activities on mathematics platforms to enrich their students' learning. Seventhly, it is vital to give the students examples from real life, other subjects, and their culture to understand the importance of solving systems of equations with three variables.

One limitation of this study is related to the group of students in the sample. All students in the sample are from the elite stream, and even if they have weak students, they are good compared with students in other groups. Also, all of them are females from the same school. Further study will include students from different streams, schools, and genders.

Conclusion

The study was conducted to figure out the difficulties faced by grade 9 students in solving systems of equations with three variables. In the first theme, the *main struggles*, the researcher found that the students struggle with basic arithmetic and algebraic skills, such as subtracting integers and solving one-step equations. For the *students' emotions* theme, the study revealed that students feel anxious and unconfident when they solve

systems of equations with three variables. The *choice of strategy* theme showed that some students could not find the best method to solve systems of equations with three variables. The last theme, *students' reflections* demonstrated that some students could not compare the systems of equations correctly. They did not know the importance of solving systems of equations with three variables. They sometimes checked their answers using substitutions, calculators, or mathematics web applications. They thought they could improve themselves by solving more systems, watching videos, or following the application steps.

References

Arcavi, A., Drijvers, P., & Stacey, K. (2017). *The learning and teaching of algebra: Ideas, insights and activities (IMPACT: Interweaving mathematics pedagogy and content for teaching)* (1st ed.) London: Routledge.

Arnawa, I. M., Yerizon, & Nita, S. (2019). Errors and misconceptions in learning elementary linear algebra. *Journal of Physics: Conference Series, 1321*(2), 022095. Retrieved from https://doi.org/10.1088/1742-6596/1321/2/022095

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning, 9*(3), 249-272. Retrieved from https://doi.org/10.1080/10986060701360910

Assadi, N., & Hibi, W. (2022). The impact of using real life situation in solving linear equations by seventh graders. *Journal of Educational and Social Research, 12*(1), 51. Retrieved from https://doi.org/10.36941/jesr-2022-0006

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *The Elementary School Journal, 84*(2), 199-212. Retrieved from https://doi.org/10.1086/461356

Birinci, D. K., Delice, A., & Aydın, E. (2014). University students' solution processes in systems of linear equation. *Procedia-Social and Behavioral Sciences, 152*, 563-568. Retrieved from https://doi.org/10.1016/j.sbspro.2014.09.244

Booth, J. L, & Koedinger, K. R. (2008). Key misconceptions in algebraic problem solving. In *Proceedings of the annual meeting of the cognitive science society* (p. 30). Retrieved from Retrieved from https://escholarship.org/uc/item/5n28t12n

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24-34. Retrieved from https://doi.org/10.1016/j.learninstruc.2012.11.002

Brink, H. I. L. (1993, May). Validity and reliability in qualitative research. *Curationis, 16*(2), a1396. Retrieved from https://doi.org/10.4102/curationis.v16i2.1396

Bybee, R. W. (2009). The BSCS 5E instructional model and 21st century skills: A commissioned paper prepared for a workshop on exploring the intersection of science education and the development of 21st century skills.

Chazan, D., Sela, H., & Herbst, P. (2012). Is the role of equations in the doing of word problems in school algebra changing? Initial indications from teacher study groups. *Cognition and Instruction, 30*(1), 1-38. Retrieved from https://doi.org/10.1080/07370008.2011.636593

DSF Literacy and Clinical Services. (2015). Understanding learning difficulties. DSF Literacy Services. Retrieved from https://uldforparents.com/wp-content/uploads/2021/01/DSF10839_Auspeld_Learning-Difficulties_PARENTS_WEB.pdf

Dumont, J. (1994). "Part 1: Theory and model." In *Learning disabilities*. Rotterdam: Lemniscaat.

Egodawatte, G. (2009). Is algebra really difficult for all students? *Acta Didactica Napocensia, 2*(4), 101-106. Retrieved from https://eric.ed.gov/?id=EJ1052268

Ernawati, E., & Muzaini, M. (2020). The analysis of students' difficulties in solving systems of linear equation problems in two variables. *Jurnal Studi Guru Dan Pembelajaran, 3*(3), 391-398. Retrieved from https://e-journal.my.id/jsgp/article/view/405/344

Fatio, N. A., Fatimah, S., & Rosjanuardi, R. (2020). The analysis of students' learning difficulties on system of linear equation in two variables topic. *Journal of Physics: Conference Series, 1521*(3), 032062. Retrieved from https://doi.org/10.1088/1742-6596/1521/3/032062

Faulkenberry, E. D., & Faulkenberry, T. J. (2006). Constructivism in mathematics education: A historical and personal perspective. *The Texas Science Teacher, 35*, 17-21. Retrieved from https://www.researchgate.net/publication/259582393_Constructivism_in_Mathematics_Education_A_Historical_and_Personal_Perspective

Ferrara, F., David, P., & Robutti, O. (2006). The role and uses of technologies for the teaching of algebra and calculus. Ideas discussed at PME over the last 30 years. In A. Gutierrez, and P. Boero (eds.), *Handbook on research on the psychology of mathematics education: Past, present and future* (pp. 237-273). Rotterdam: Sense Publishers. Retrieved from https://doi.org/10.1163/9789087901127_010

Heirdsfield, A. (2002). The interview in mathematics education: The case of mental computation. In *Proceedings annual conference of the Australian association for research in education 2002*, Brisbane, Australia. Retrieved from https://eprints.qut.edu.au/1055/1/http___www.aare.edu.au_02pap_hei02334.htm.pdf

Heng, M. A., & Sudarshan, A. (2013). "Bigger number means you plus!"—Teachers learning to use clinical interviews to understand students' mathematical thinking. *Educational Studies in Mathematics, 83*(3), 471-485. https://doi.org/10.1007/s10649-013-9469-3

Hsiao, H. S., Lin, C. Y., Chen, J., & Peng, Y. F. (2017). The influence of a mathematics problem-solving training system on first-year middle school students. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(1), 77-93. Retrieved from https://doi.org/10.12973/ejmste/77902

Hudson, S., Kadan, S., Lavin, K. T., & Vasquez, T. (2010). Improving basic math skills using technology. Saint Xavier University. Retrieved from https://files.eric.ed.gov/fulltext/ED512698.pdf

Issakova, M. (2006). Comparison of student errors made during linear equation solving on paper and in interactive learning environment. University of Tartu, Institute of Computer Science.

Ives, B., & Hoy, C. (2003). Graphic organizers applied to higher-level secondary aathematics. *Learning Disabilities Research and Practice, 18*(1), 36-51. Retrieved from https://doi.org/10.1111/1540-5826.00056

Jacobs, G. J., & Durandt, R. (2014). Mathematics students teachers' approach to, experience of and attitudes towards mathematical modeling. In *Proceedings of 5th ISTE (international science, technology and mathematics) education conference*, Limpopo, South Africa. Retrieved from https://doi.org/10.13140/2.1.3368.4481

Jacobs, G. J., & Spangenberg, E. D. (2014). Mathematics teachers' attitudes towards the subject: The influence of gender, age and teaching experience. In *Proceedings of the ISTE 5th international science, technology and mathematics education conference* (pp. 91-100), Limpopo, South Africa. Retrieved from https://uir.unisa.ac.za/bitstream/handle/10500/22912/Gerrie%20J%20Jacobs%2C%20Erica%20Spangenberg.pdf?sequence=1&isAllowed=y

Jazim, J., Anwar, R. B., & Rahmawati, D. (2017). The use of mathematical module based on constructivism approach as media to implant the concept of algebra operation. *International Electronic Journal of Mathematics Education, 12*(3), 579-583. Retrieved from https://doi.org/10.29333/iejme/634

Jenkins, O. F. (2010). Developing teachers' knowledge of students as learners of mathematics through structured interviews. *Journal of Mathematics Teacher Education, 13*(2), 141-154. Retrieved from https://doi.org/10.1007/s10857-009-9129-9

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317-326. Retrieved from http://www.jstor.org/stable/3482333

Larino, L. B. (2018). An analysis of errors made by grade 7 students in solving simple linear equations in one variable. *International Journal of Scientific & Engineering Research, 9*(12), 764-769. Retrieved from https://doi.org/10.14299/ijser.2018.12.05

LeCompte, M. D., & Goetz, J. P. (1982). Problems of reliability and validity in ethnographic research. *Review of Educational Research, 52*(1), 31-60. Retrieved from https://doi.org/10.3102/00346543052001031

Lewis, K. E., & Fisher, M. B. (2017). Clinical interviews: Assessing and designing mathematics instruction for students with disabilities. *Intervention in School and Clinic, 53*(5), 283-291. Retrieved from https://doi.org/10.1177/1053451217736864

Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on math anxiety. *Psychology Research and Behavior Management, 11*, 311-322. Retrieved from https://doi.org/10.2147/PRBM.S141421

Magruder, R. (2012). Solving linear equations: A comparison of concrete and virtual manipulatives in middle school mathematics (Ph.D. thesis, University of Kentucky, 2012). Retrieved from https://uknowledge.uky.edu/edc_etds/2

Maguire, M., & Delahunt, B. (2017). Doing a thematic analysis: A practical, step-by-step guide for learning and teaching scholars. *AISHE-J, 9*(3), 3351-33514. Retrieved from https://ojs.aishe.org/index.php/aishe-j/article/view/335Mc

McDonough, A., Clarke, B., & Clarke, D. M. (2002). Understanding, assessing and developing children's mathematical thinking: The power of a one-to-one interview for preservice teachers in providing insights into appropriate pedagogical practices. *International Journal of Educational Research, 37*(2), 211-226. Retrieved from https://doi.org/10.1016/s0883-0355(02)00061-7

Mevarech, Z. R., & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning. *British Journal of Educational Psychology, 73*(4), 449-471. Retrieved from https://doi.org/10.1348/000709903322591181

Mullis, I. & Martin, M. (2016). *TIMSS 2015 assessment frameworks*. Chestnut Hill: TIMSS & PIRLS, International Study Center. Retrieved from https://timssandpirls.bc.edu/timss2015/downloads/T15_Frameworks_Full_Book.pdf

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). TIMSS 2019 international results in mathematics and science. TIMSS & PIRLS International Study Center. Retrieved from https://timssandpirls.bc.edu/timss2019/international-results/

Mushtaq, F. (2013). Mathematics achievements among high school students in Afghanistan: Gender differences in math achievements (M.Sc. thesis, Karlstad University, 2013). Retrieved from http://www.diva-portal.org/smash/get/diva2:695182/FULLTEXT01.pdf

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington D.C.: National Academy Press. Retrieved from https://doi.org/10.17226/9822

Novianti, N., & Priatna, B. A. (2019). Students' difficulties analysis in solving systems of linear equations in two variables. In *Proceedings of 1st international seminar STEMEIF (science, technology, engineering and mathematics learning international forum): "Strengtheing the STEM education and digital skills"*.

OECD. (2018). PISA 2021 mathematics framework (SECOND DRAFT). Retrieved from https://www.oecd.org/pisa/pisaproducts/pisa-2021-mathematics-framework-draft.pdf

Rahman, M. S. (2016). The advantages and disadvantages of using qualitative and quantitative approaches and methods in language "testing and assessment" research: A literature review. *Journal of Education and Learning, 6*(1), 102-112. Retrieved from https://doi.org/10.5539/jel.v6n1p102

Selltiz, C., Wrightsman, L. C., & Cook, S. W. (1976). *Research methods in social relations* (3rd ed.). Holt: Rinehart & Winston.

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students (NCEE 2014-4333)*. Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from http://whatworks.ed.gov

Steenbrugge, H., Valcke, M., & Desoete, A. (2010). Mathematics learning difficulties in primary education: Teachers' professional knowledge and the use of commercially available learning packages. *Educational Studies, 36*(1), 59-71. Retrieved from https://doi.org/10.1080/03055690903148639

Sulistiawati, & Surgandini, A. (2019). Investigating the learning obstacle and the self-confidence of students college in material understanding ability of linear algebra course. *International Journal of Scientific & Technology Research, 8*(9), 405-411. Retrieved from http://www.ijstr.org/final-print/sep2019/Investigating-The-Learning-Obstacle-And-The-Self-Confidence-Of-Students-College-In-Material-Understanding-Ability-Of-Linear-Algebra-Course.pdf

Tambychik, T., & Meerah, T. S. M. (2010). Students' difficulties in mathematics problem-solving: What do they say? *Procedia-Social and Behavioral Sciences, 8*, 142-151. Retrieved from https://doi.org/10.1016/j.sbspro.2010.12.020

Tezer, M., & Cumhur, M. (2017). Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects. *EURASIA Journal of Mathematics, Science and Technology Education, 13*(8), 4789-4804. Retrieved from https://doi.org/10.12973/eurasia.2017.00965a

Turan, S., & Matteson, S. M. (2021). Middle school mathematics classrooms practice based on 5E instructional model. *International Journal of Education in Mathematics, Science and Technology, 9*(1), 22-39. Retrieved from https://doi.org/10.46328/ijemst.1041

Vlassis, J. (2001). Solving equations with negatives or crossing the formalizing gap. In M. van den Heuvel-Panhuizen (ed.), *Proceedings of the twenty-fifth international conference for the psychology of mathematics education* (Vol. IV, pp. 375-382), Utrecht, The Netherlands.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in "negativity". *Learning and Instruction, 14*(5), 469-484. Retrieved from https://doi.org/10.1016/j.learninstruc.2004.06.012

Warren, E., Trigueros, M., & Ursini, S. (2016). Research on the learning and teaching of algebra. In Á. Gutiérrez, G. C. Leder, & P. Boero (eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 73-108). Rotterdam: Sense Publishers. Retrieved from https://doi.org/10.1007/978-94-6300-561-6_3

Wiberg, M. (2019). The relationship between TIMSS mathematics achievements, grades, and national test scores. *Education Inquiry, 10*(4), 328-343. Retrieved from https://doi.org/10.1080/20004508.2019.1579626