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Analysis of a Simple RC Structure with "Plastic Hinges" Considering Crack Development

Irina Kerelezova[1] and Emre Yusuf[2]

1. University of Architecture, Civil Engineering and Geodesy, Sofia BG-1046, Bulgaria

2. Building Management and Consultancy Company, Sofia BG-1046, Bulgaria

Abstract: In the present study an idea of combination of fracture mechanics and Limit State Analysis is presented. The model of the "plastic hinges" is made by using theory of fracture mechanics. Analysis of a simple RC (reinforced concrete) structure by means of fracture mechanics is performed by taking into account crack development. Reinforcement plastification and nonlinear behavior of concrete are considered in the numerical model. Comparisons with a simple Limit State Analysis plastic hinge model are made. The numerical analysis is made by using ANSYS software product. This research is part of a project aimed at determining the maximum bending moment in a beam subjected to bending with the methods of fracture mechanics and the definition of a simplified formula specifying the one obtained by the Limit State Analysis.

Key words: Limit State Analysis, fracture mechanics, RC-structure, ANSYS.

1. Introduction

The theory of Limit State Analysis is simple and very useful in practice for determining the plastic moment in structures subjected in bending. For materials like a steel, it is clear and simple to define and calculate the plastic moment but for the materials like concrete it is more complicate. Parameters as a reinforcement ratio, concrete class and loads rate are important and influence over the plastic moment. In this case Eurocode gives formulas and procedures to define the plastic moment. It is well known that the concrete has very low resistance in tension, and this is the main reason in the theory of Limit State Analysis to neglect it. This consideration leads to simplicity and results good enough. Another theory concerning concrete shows impotency and significancy of its behavior in tension, namely the Fracture Mechanics theory. According to this theory the tensile strength of the concrete has a significant part in its behavior not only in shear but also in bending. Logically one may ask how the plastic moment will change if the tensile strength is taken into account and even more how the cracking process in time influences plastic moments and the carrying capacity of the structure.

Presented study is a part of a project aimed to determine the maximum bending moment in a beam subjected to bending with the methods of fracture mechanics and the definition of a simplified formula specifying the one obtained by the Limit State Analysis. In previous research of the authors—see Refs. [1-3] the influence of some parameters on the so-called maximum moment (plastic moment) calculated by taking into account the cracking process in the element has been investigated. The main result obtained in these studies is that the maximum moment is not influenced by the concrete class which is different from Eurocode. Another question posed there is the influence of the fracture energy which is not considered in Eurocode. From the literature one may see that the fracture energy is almost a constant for concretes with normal strength [4]. This means that this is not a factor for obtaining results in Ref. [1]. Other results from Refs. [2, 3] show the

Corresponding author: Irina Kerelezova, Acoss. Prof. PhD, research fields: fracture mechanics.

influence of the numerical model and the crack development process in time. For these reasons the authors of the present study focused on the crack development process and the caring capacity of a structure. To investigate the load caring capacity and cracking process of a structure, more than one plastic hinge is chosen.

2. Numerical Modelling and Initial Solution

2.1 Finite Element Model

For present numerical modelling Concrete 65 finite elements in ANSYS program is used. For better understanding some information for this element will be presented.

The implementation of nonlinear material laws is employed in ANSYS, where a special three-dimensional eight nodes solid isoparametric element, Solid 65 is developed. It models the nonlinear response of quasi-brittle materials and is based on a constitutive model for the triaxial behavior of concrete after Williams and Warnke—see Fig. 1 and Ref. [5] for details.

The element includes a smeared crack idea for handling cracking in tension zones and a plasticity algorithm to account for the possibility of concrete crushing in compression zones. The element has one solid material and up to three rebar materials which are input as real constants with the orientation angles. The rebars are positioned in the centre of the element as one may see in Fig. 2. These real constants include material number and the volume ratio as the rebar volume divided by the total element volume.

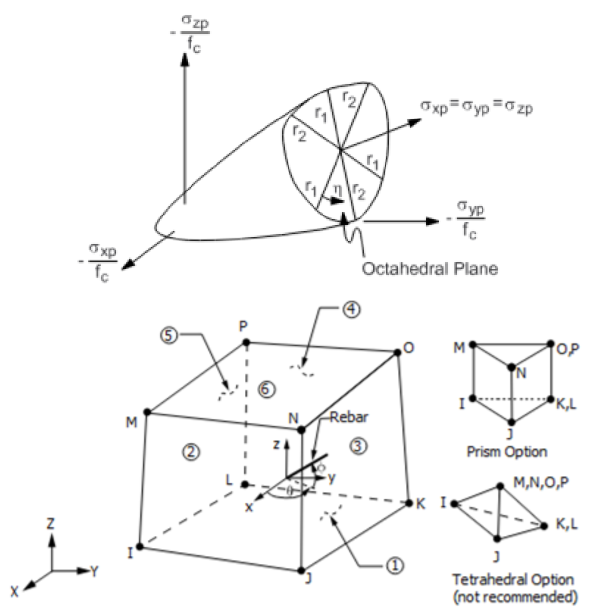

Fig. 1 Williams and Warnke physical model and Concrete 65 FE in ANSYS.

The shear transfer across a crack can be varied between full shear transfer and no shear transfer at a cracked section. In ANSYS that option is controlled by two coefficients—β_1 for an open crack and β_2 for a closed crack. They are so-called shear retention factors and range from 0 (no aggregate interlock) to 1 (full aggregate interlock) (see Ref. [5]). For the present simulation the values of 0.6 and 1.0 for β_1 and β_2 respectively are chosen to minimise the convergency problems (see Ref. [6]). The stress relaxation is used only to accelerate convergence of the calculations and a multiplier for tensile stress relaxation is taken to be 1.0.

It is important to note that in this model fracture energy is not considered but it is a very good model to represent crack development in the reinforced concrete.

2.2 Choice of a Beam Size

It is well known the size effect phenomena for concrete and reinforced concrete and its influence on the behavior of such structures. For this reason, the sizes of the beam to investigate are important points for choice of the research. In previous works of the authors this phenomenon has appeared as a problem. The chosen beam was with very brittle behavior and partly with high shear cracks development. To avoid this phenomenon and to choose proper dimensions for the present study an initial solution of a simple beam with chosen material characteristics is performed. Here will be presented shortly the results for the simple beam with chosen dimensions. The aim of the choice is to have quasi-brittle or plastic behavior of the beam and without shear cracks development. To easily obtain plastification in the reinforcement very low reinforcement ratio is chosen.

The dimensions of the simple beam are presented in Fig. 2.

A concrete material C35/40 according to Eurocode is chosen with modules of elasticity E = 34,000 MPa, Poisson ratio ν = 0.2, tensile strength f_t = 3.2 MPa and compressive strength f_c = 35 MPa. For the reinforcement B420 is chosen with f_y = 420 MPa. All dimensions of the beam are in function of its high D = 500 mm as it is shown in Fig. 2. From Figs. 3-6 the main results are shown.

From the results shown in Figs. 3-6, the following main conclusions are drawn:

(1) The beam behaves on pure bending without the development of shear cracks which may be seen in Fig. 4.

(2) The two sides of the beam on left and on right of the plastic hinge behaves almost as infinite rigid body and the relative rotation is concentrated in the process zone—see Fig. 3.

(3) The plastification of the reinforcement is reached for almost full width of the beam (Fig. 5). Further plastification is possible with a more accurate solution but for the aim of present study, it is enough good. The reason to stop at the present point is that the solutions are very sensitive and hardly converged.

(4) From the load-deflection curve one may see that the beam's behavior is not brittle—Fig. 6. At the moment of reinforcement plastification (the maximum load caring capacity) there is a plateau—the maximum load keeps for a while before the solution stops. This plateau is very short because of the end of the solution which is caused by the convergency problems as it was mentioned in the previous point here.

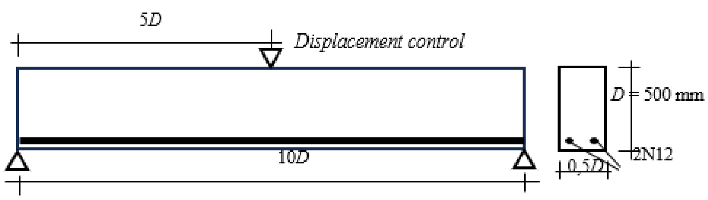

Fig. 2 Dimensions and statical scheme of a simple beam.

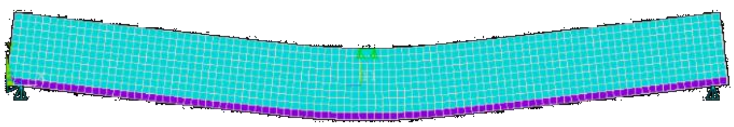

Fig. 3 Deformed chape and finite element model of the simple beam.

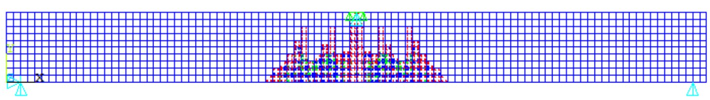

Fig. 4 Cracking process zone of the beam.

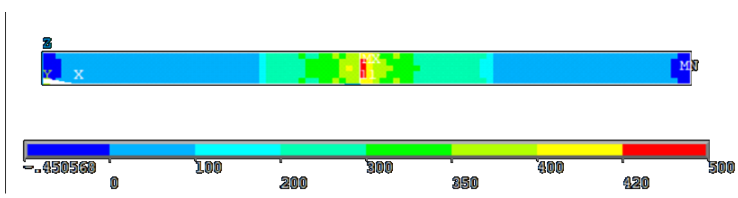

Fig. 5 Stresses in the reinforcement layer of the beam.

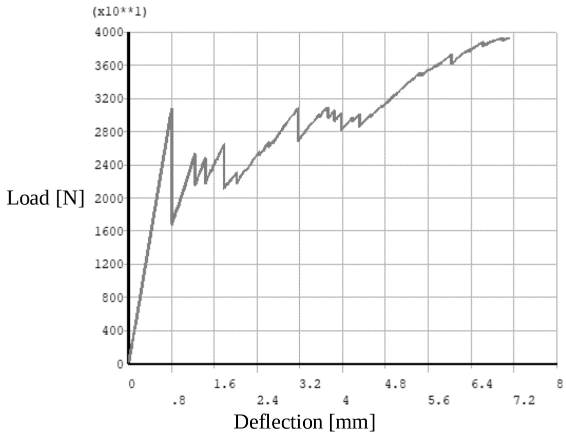

Fig. 6 Load-deflection curve of the simple beam.

As an additional conclusion can be noticed the load caring capacity value of the simple beam is 39.3 kN. For comparison this calculated by using Limit State Analysis theory is 35.5 kN. As it is expected, the load caring capacity obtained from the nonlinear solution is much bigger than this obtained from the simple theory. The reason for this is the crack development process. This shows that it is very important to take into consideration the crack development process in the load caring capacity calculations.

From the presented solution a final conclusion may be made: the dimensions and material data of the simple beam (Fig. 2) are suitable for the present study and will be used for the next step of the study.

For the main research a structure with more than one plastic hinge will be used. It is convenient for its behavior to be in pure bending as it was mentioned above (without development of shear cracks).

3. Results, Comparisons and Discussions regarding the Simple Structure

The beam from Fig. 2 will be transformed into a simple structure with two expected plastic hinges—presented in Fig. 7.

A concrete material C35/40 according to Eurocode is chosen with modulus of elasticity E = 34,000 MPa, Poisson ratio ν = 0.2, tensile strength f_t = 3.2 MPa and compressive strength f_c = 35 MPa. For the reinforcement B420 is chosen with f_y = 420 MPa.

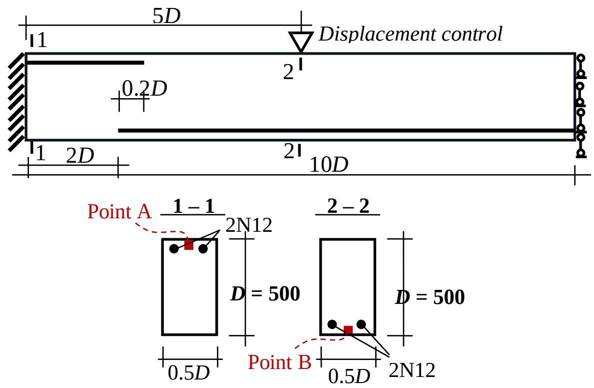

Fig. 7 Dimensions and statical scheme of a simple structure.

Before the numerical nonlinear simulation, a simple solution by using Limit State Analysis [7] is performed and the result is presented below—see Figs. 8 and 9. The plastic moment is calculated for the reinforced concrete cross section according to the Eurocode [8].

The 3D finite element model with real constants of the elements used for the nonlinear solution is shown in Fig. 10.

A complex of results curves is presented in Fig.11. As the solution of the structure is performed in displacement control all results are in dependence of the structural deflection. The grey line represents the load force corresponding to the deflection. For better visibility the scale of 10*kN is chosen for its plot.

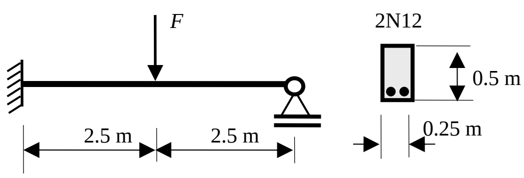

Fig. 8 Statical scheme and cross section for the Limit State Analysis solution.

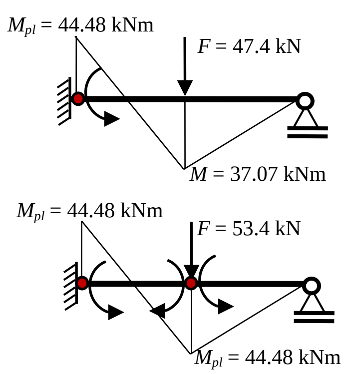

Fig. 9 Limit State Analysis results.

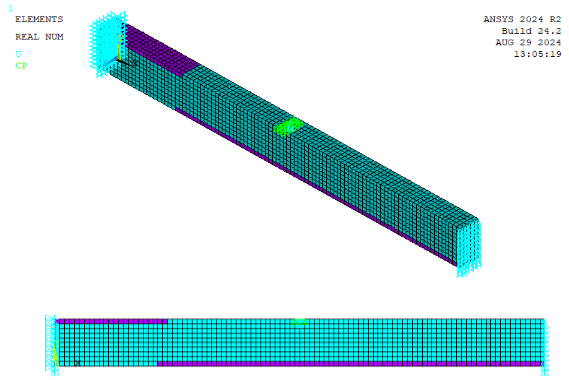

Fig. 10 3D model with the boundary conditions and the real constants of the elements.

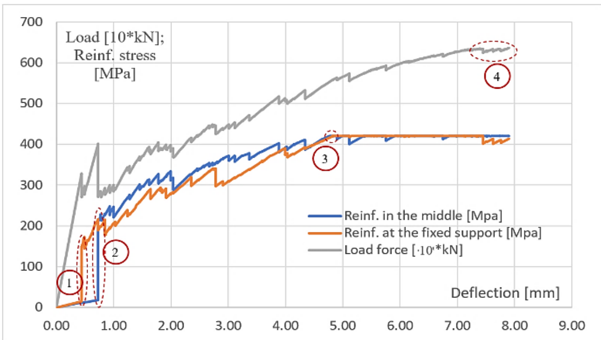

Fig. 11 3D model with real constants of the elements.

Orange curves represent the reinforcement stress at Point A (see Fig. 7). The blue curve represents the reinforcement stress in Point B (see Fig. 7). In the 3D model the reinforcement is modeled as a distributed (see Fig. 10). The cross section of the reinforcement is divided by the cross section of the finite elements in the bottom layer for the middle of the beam and by the cross section of the finite elements in the top layer for the fixed support point. For this reason, in Point A (middle of the cross section next to the fixed support) the finite element has reinforcement according to the real constants, similarly for Point B which is in the middle of the cross section under the load point of the structure.

From the results shown in Fig. 11 it is easy to follow the processes in the structure. There are four important moments in its behavior. Moment 1 and Moment 2 show the initiation of the crack’s development in the structure—see Fig. 12. Logically, because of the cracks opening there is a jump in the reinforcement stresses and the load force too. These moments cannot be considered as a plastic hinge because the reinforcement stress is far from plastification but shows the order of its development—first crack is next to the fixed support and second is in the middle of the structure.

Next significant point is Moment 3 in Fig. 11 which shows the moment of first plastification. In Fig. 13, the plastification zones on the top and bottom side of the structure are shown. From this figure it is clear that near the fixed support the full cross section has plasticized but in the middle bottom side (in Point A) just only one element has plastification. This moment can be noticed as a first “plastic hinge” appearance. The load force at this moment is 56 kN which is around 18% higher than this calculated by using the Limit State Analysis value (see Fig. 9).

Last significant point of the structural behavior is the final plastification of the structure—Moment 4 in Fig. 11. At this moment the load force remains almost constant and the stress at the elements in Point A and Point B remains plasticized. The zones of plastification in Moment 4 are shown in Fig. 14. As one may see, these zones are bigger than these on Moment 3.

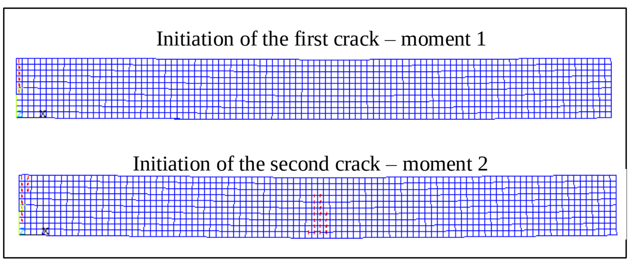

Fig. 12 Initiation of the cracks at the beginning of the loading.

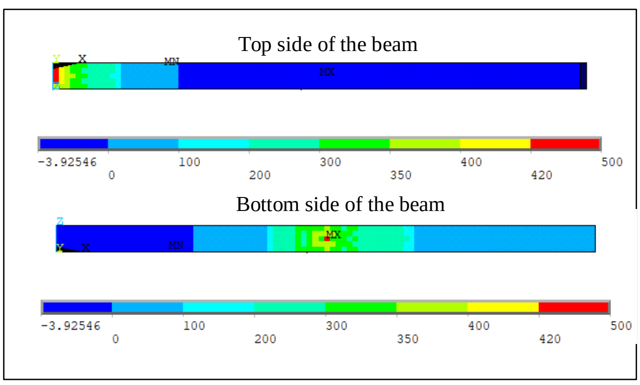

Fig. 13 First plastification moment.

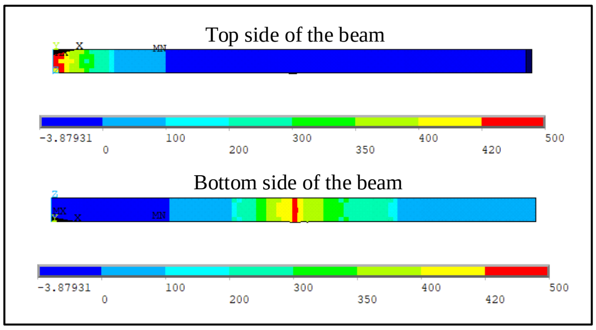

Fig. 14 Plastification zones at the end of the solution.

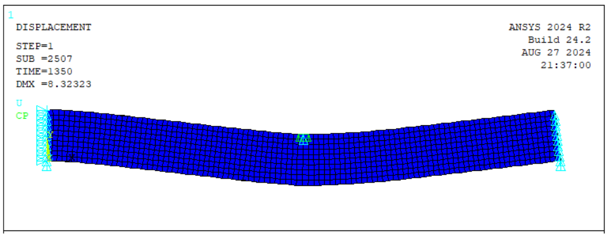

Fig. 15 Deformed shape in the final stage of the solution.

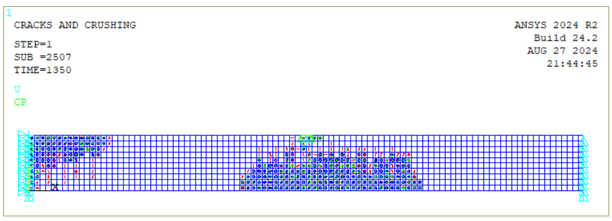

Fig. 16 Cracks distribution in the final stage of the solution.

This moment is considered as the appearance of the second and final "plastic hinge" for the structure. It is seen in Fig. 14 that at this moment the full width of the cross section in the middle of the beam is plasticized. The load force at this moment is considered as load caring capacity and its value is 63.7 kN which is nearly 20% higher than this calculated with the simple theory of the Limit State Analysis represented in Fig. 9. The deformed shape and the cracks distribution at Moment 4 are shown in Figs. 15 and 16.

On present nonlinear solution, it is difficult to distinguish the two "plastic hinges" that is why in the present study the moment of the full cross section width plastification is assumed as a criterion for "plastic hinge" appearance but actually the "plastic hinge" formation starts with the first's cracks initiation. By using present assumption it may be noticed that at the current nonlinear simulation the two hinges appear in order corresponding to the theory.

4. Conclusions

From previous paragraph and paragraph 2 solutions, the following conclusions may be noticed:

- Presented 3D model is convenient for determination of the load caring capacity of elementary reinforced concrete structures;
- The width of the "plastic hinge" (cracking process zone) corresponds to the theoretical value of the Limit State Analysis (around 1/3 of the beam span);
- The order of the appearance of the plastic hinges for the elementary structure with more than one hinge corresponds to the theory;
- The value of the load caring capacity of structures with only one plastic hinge is around 10% higher than this calculated by using the simple theory of Limit State Analysis;
- The value of the load caring capacity of structures with more than one plastic hinge is around 20% higher than this calculated by using the simple theory of Limit State Analysis.

Last two points of the concluding remarks show the impotency of the consideration of the cracking process in time to calculate the load caring capacity and the "plastic" (maximum) moment of the cross section by using fracture mechanics theory.

Acknowledgments

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References

[1] Yusuf, E., and Kerelezova, I. 2024. "Parametric Study of the Bending Moment in a Cracked RC Beam." *Engineering Sciences* 61 (January): 30-40.

[2] Kerelezova, I. 2024 "Analysis of the Maximum Bending Moment in a Beam by Using Smeared Crack Method." *Annuaire de L'universite D'architecture, de Genie Civil et de Geodesie-Sofia* 57 (July): 869-78. (in Bulgarian)

[3] Kerelezova, I. 2024. *Determination of a Limit (Plastic)*

Moment of a Beam Subjected on Bending by Using Fracture Mechanics Methods. Technical Report of a Project BH 275/23, Research, Consultancy and Design Centre, UACEG. (in Bulgarian)

[4] Darwin, D., Barham, S., Kozul, R., and Luan, S. 2001 “Fracture Energy of High-Strength Concrete.” *ACI Materials Journal* 98 (September-October): 410-7.

[5] ANSYS, Inc. 2024. “Ansys® Academic Research Mechanical, Release 2024 R1, Help System, Coupled Field Analysis Guide.”

[6] Parvanova, S., Kazakov, K., Kerelezova, I., Gospodinov, G., and Nielsen, M. P. 2005. “Modelling the Nonlinear Behavior of R/C Beams with Moderate Shear Span and without Stirrups Using ANSYS.” In *Proceedings of the Conference VSU’ 2005*, Sofia, pp. 65-70.

[7] Nielsen, M. P. 1999. *Limit Analysis and Concrete Plasticity*. Boca Raton: CRC Press.

[8] EN 1992-1-1 Eurocode 2. 2002, “Part I: General Rules and Rules for Buildings.” In *Design of Concrete Structures*. Brussels: European Committee for Standardization.